Exponentials & Logarithms
Concept Review

I. **Basic Log/Exponent Rules** (x, y, b all positive, b ≠ 1)

(1)	$\log_b(xy) = \log_b x + \log_b y$	**(Log of a Product Rule)**
(2)	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	**(Log of a Quotient Rule)**
(3)	$\log_b(x^k) = k\log_b x$ (any k)	**(Log of a Power Rule)**
(4)	$\log_b x = \frac{\log_c x}{\log_c b}$ (for any $c > 0, c \neq 1$)	**(Change of Base Rule)**

A logarithm and an exponent with the same base are <u>**inverse**</u> functions. This means that:

(5) **(Log/Exponent Inverse Rule)**

$$y = \log_b x \Leftrightarrow b^y = x \quad \text{and}$$

$$\log_b(b^x) = b^{\log_b x} = x \quad \text{and,}$$

as a specific example, $\ln(e^x) = e^{\ln x} = x$

In fact, all of the above rules are true for the specific case when the base is e, so the above rules could all be written with "ln" in place of "$\log_b$".

EXERCISES: ***Solve each of the following equations for a in terms of b.***

1) $e^{3a+5} = b$

$3a+5 = \ln b$

$a = \boxed{\frac{\ln b - 5}{3}}$

2) $\ln 3a - \ln 5b = 12$

$\ln\frac{3a}{5b} = 12$

$\frac{3a}{5b} = e^{12}$

$3a = 5be^{12}$

$a = \boxed{\frac{5be^{12}}{3}}$

3) $\ln(4a-3) = b$

$4a-3 = e^b$

$4a = 3+e^b$

$a = \boxed{\frac{3+e^b}{4}}$

4) $\ln(3a^4) = b + \ln a^3$

$\ln(3a^4) - \ln a^3 = b$

$\ln\left(\frac{3a^4}{a^3}\right) = b$

$\ln 3a = b$

$3a = e^b$

$a = \boxed{\frac{e^b}{3}}$

5) $a^{17} = b$

$a = \boxed{\sqrt[17]{b}}$ or $\boxed{b^{1/17}}$

6) $be^a = 7$

$e^a = \frac{7}{b}$

$a = \boxed{\ln\frac{7}{b}}$ or $\boxed{\ln 7 - \ln b}$

7) $\log_b(a^5) = 60$

$5\log_b a = 60$

$\log_b a = 12$

$\boxed{a = b^{12}}$

8) $a^3 = 8b^{10}$

$a^3 = 2^3 b^{10}$

$a = \boxed{2b^{10/3}}$

9) $\log_a 12 = b$

$a^b = 12$

$a = \boxed{\sqrt[b]{12}}$ or $\boxed{12^{1/b}}$

10) $b^a = 36$

$\log(b^a) = \log 36$

$a\log b = \log 36$

$a = \boxed{\frac{\log 36}{\log b}}$ or $\boxed{\log_b 36}$