Honors Precalculus
Assorted Review §§ 6.1 – 6.3 (calculators OK)

Name KEY
Period _____

Show All Work!

1. Let $u = \langle -5, 3 \rangle$ and $v = \langle 2, -1 \rangle$

a) Find $|u|$ and the direction angle of u.

$|u| = \sqrt{(-5)^2 + 3^2} = \sqrt{25+9} = \boxed{\sqrt{34}}$

$\theta = \tan^{-1}\left(\frac{3}{-5}\right) + 180$

$= \boxed{149.036^\circ}$

b) Find $u + 5v$

$= \langle -5, 3 \rangle + \langle 10, -5 \rangle$

$= \boxed{\langle 5, -2 \rangle}$

c) Find a unit vector that has the same direction as v.

$\frac{u}{|u|} = \frac{\langle 2, -1 \rangle}{\sqrt{2^2 + (-1)^2}} = \boxed{\left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle}$

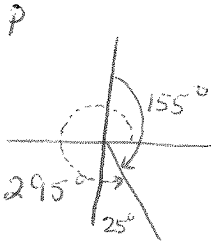
d) Find the dot product $u \cdot v$

$(-5)(2) + (3)(-1) = -10 - 3 = \boxed{-13}$

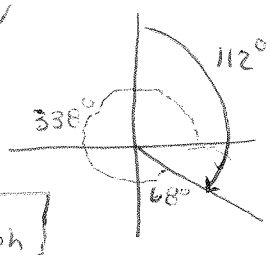
e) Find the measure of the angle between u and v.

$\cos^{-1}\left(\frac{u \cdot v}{|u|\,|v|}\right) = \cos^{-1}\left(\frac{-13}{\sqrt{34}\sqrt{5}}\right) = \boxed{175.601^\circ}$

2. A plane is flying at an air speed of 520 mph, heading towards a bearing of 155°. There is a 62 mph wind with a bearing of 112°.



a) Find the component form of actual velocity of the airplane.



$P + W = \boxed{\langle 277.247, -494.506 \rangle}$ (a, b)

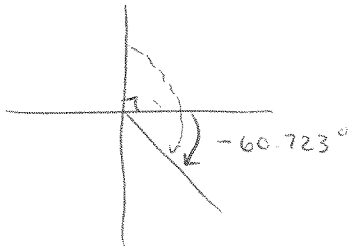
b) Find the ground speed of the airplane.

$|P + W| = \sqrt{a^2 + b^2} = \boxed{566.923 \text{ mph}}$

$P = \langle 520\cos 295^\circ, 520\sin 295^\circ \rangle$
$W = \langle 62\cos 338^\circ, 62\sin 338^\circ \rangle$

c) Find the actual direction (as a bearing) of the plane's travel.

$\theta = \tan^{-1}\left(\frac{-494.506}{277.247}\right) = -60.723^\circ$ (trig angle)



$= \boxed{150.723^\circ}$ (bearing)

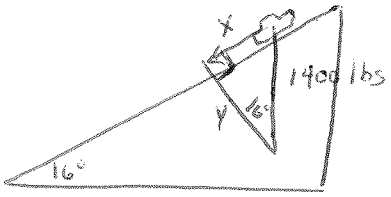
3. A car weighing 1400 lbs is parked on a hill with an incline of 16°, and the parking brake is not engaged.

a) Find the amount of force that would be required to prevent the car from rolling down the hill.

$\sin 16^\circ = \frac{x}{1400}$

$x = 1400 \sin 16^\circ$

$= \boxed{385.892 \text{ lbs}}$



b) Find the amount of force that is being exerted perpendicular to the hill.

$\cos 16^\circ = \frac{y}{1400}$

$y = 1400 \cos 16^\circ$

$= \boxed{1,345.766 \text{ lbs}}$

4. Eliminate the parameter t and identify the graph (be specific):

$x = 5\cos 2t, \quad y = 5\sin 2t, \quad 0 \le t \le \frac{\pi}{2}$

$x^2 = 25\cos^2 2t$

$y^2 = 25\sin^2 2t$

$x^2 + y^2 = 25(\cos^2 2t + \sin^2 2t)$

$\boxed{x^2 + y^2 = 25 \quad \text{(top half)}}$

$0 \le t \le \frac{\pi}{2}$

$0 \le 2t \le \pi \rightarrow y \ge 0$ since sine is positive on this interval

semicircle - top half of circle centered at origin with radius 5

5. Write a parametric equation for

a) a line segment with endpoints $(5,-2)$ and $(-2,3)$.

$x = \boxed{7}t + \boxed{-2}$

$y = \boxed{-5}t + \boxed{3}$

t=1 t=0

run = 7

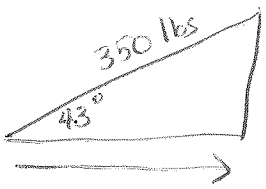
rise = -5

$x = 7t - 2$
$y = -5t + 3$
$0 \le t \le 1$

b) a circle with radius 6 and center $(5,-2)$.

$x = 6\cos T + 5$
$y = 6\sin T - 2$
$0 \le T \le 2\pi$ ← (optional - not restricting t would allow it to retrace the same path)

6. A force of 350 pounds is being used to drag a sandbag at the end of a rope. If the rope is at a 43° angle to the horizontal, and if the sandbag is dragged 22 feet, how much work has been done in moving the sandbag?



Force in horizontal direction

$= 350 \cos 43^\circ$

$= 255.974 \text{ lbs}$

Work = (Force)(Distance)

$= (255.974)(22)$

$= \boxed{5,631.424 \text{ ft-lbs}}$

7. A football is kicked from ground level with an initial velocity of 70 ft/sec at an angle of elevation of 60°.

a) When will the ball hit the ground?

$X = (70\cos 60^\circ)t$

$Y = -16t^2 + (70\sin 60^\circ)t$

$Y = 0$

$-16t^2 + (70\sin 60^\circ)t = 0$

$t(-16t + 70\sin 60^\circ) = 0$

$t = 0$ or $70\sin 60^\circ = 16t$

$t \approx 3.789$

AFTER $\approx$ 3.789 SECONDS

b) How far away (horizontally) will the ball be when it hits the ground?

$X = (70\cos 60^\circ)(3.789) \approx$ 132.610 ft

c) What is the maximum height of the ball?

VERTEX of PARABOLA $At^2 + Bt + C = 0$ is at $t = \frac{-B}{2A}$

$t = \frac{-70\sin 60^\circ}{2(-16)} \approx 1.894$ (substitute into the Y-equation)

$Y(1.894) \approx$ 57.422 ft

8. A baseball is hit with an initial velocity of 150 ft/sec from 3 feet above level ground at a 20° angle. A 20 foot high fence is 400 feet from the home plate.

a) Write parametric equations to track the horizontal and vertical position of the ball at time t.

$X = (150\cos 20^\circ)t$

$Y = -16t^2 + (150\sin 20^\circ)t + 3$

b) Will the ball go over the wall? Describe specifically what happens – for example, "the ball clears the fence by 5.24 feet" or "the ball hits the wall at a height of 7.31 feet" or "the ball falls 23.74 feet short of reaching the fence" SHOW WORK to justify your conclusion!

$(150\cos 20^\circ)t = 400$

$t \approx 2.838$

$Y(2.838) \approx 19.738$

the ball hits the wall at a height of approximately 19.738 ft

9. Stanley and Livingstone are competing in a 250-foot race. Stanley (who can sprint at a rate of 23 feet per second) lines up 25 feet behind the starting line in order to give Livingstone (who can sprint at a speed of 21 feet per second) a head start. Who wins the race, and by how many feet?

STANLEY $X = 21t$, $Y = 5$

LIVINGSTONE $X = 23t - 25$, $Y = 3$

IT WOULD TAKE LIVINGSTONE 11.956 SECONDS TO FINISH, STANLEY FINISHES IN 11.905, SO HE WINS!

$23t - 25 = 250$

$23t = 275$

$t \approx 11.956$ seconds

$21t = 250$

$t \approx 11.905$ seconds

LIVINGSTONE'S POSITION AT THIS TIME IS $23(11.905) - 25 = 248.810$ ft, SO STANLEY WON BY 1.190 FEET

Precalculus Honors
6.3 Day 3 – Simulations using parametric equations

1) Casey can sprint at the rate of 28 feet/second and Danielle sprints at 26 feet/second. Casey gives Danielle a 10 foot head start (Casey starts 10 feet behind the starting line).

a) Write parametric equations that can be used to model a race. (can you create a finish line?)

CASEY $X_1 = 28T - 10$, $Y_1 = 1$

DANIELLE $X_2 = 26T$, $Y_2 = 2$

FINISH LINE $X_3 = 400$, $Y_3 = 50T$

b) Who wins a 400-ft race and

CASEY

c) ...by how many feet?

$28T - 10 = 400$

$28T = 410$

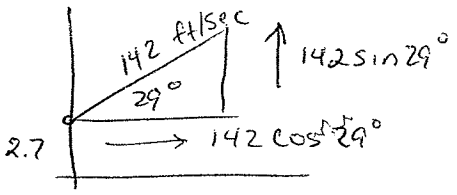
$T = \frac{410}{28} \approx 14.643$ sec

DANIELLE IS AT ≈ 380.71 @ $t = 14.643$ SECONDS,

SO CASEY WINS BY $\approx$ 19.29 feet

2) Rufus hits a baseball towards an 18-foot high fence that is 420 feet from home plate. The ball is hit when it is 2.7 feet above the ground with an initial velocity of 142 ft/sec and leaves the bat at an angle of 29° with the horizontal.

a) Write the equations that model the path of the baseball.



$X_1 = (142\cos 29°)T \approx 124.20t$

$Y_1 = -16t^2 + (142\sin 29°)t + 2.7$

$X_1 \approx 124.20t$

$Y_1 \approx -16t^2 + 68.84t + 2.7$

b) How long will it take until the ball reaches the wall?

$t \approx$ 3.382 sec

$420 = 142\cos 29° T$

$T = \frac{420}{142\cos 29°} \approx 3.382$ sec

c) Does the ball go over the wall? If so, by how much?

$Y(3.382 \text{ sec}) = 52.53$ feet $\rightarrow$

$52.53 - 18 = 34.53$ ft

YES — clears wall by $\approx$ 34.53 feet