Precalculus Honors
Review 9.1, 9.2, 10.1, 10.4

Name KEY

1. A coin is tossed, a die is rolled, and a card is drawn from a deck. How many possible outcomes does this experiment have?

$2 \cdot 6 \cdot 52 = \boxed{624}$

2a. How many different two-element subsets does the set {A, E, I, O, U} have?

$_5C_2 = \boxed{10}$

b. How many different two-letter "words" can be made using the letters from the set in part 'a'?

$_5P_2 = \boxed{20}$

3. An airline company overbooks a particular flight and seven passengers are "bumped" from the flight. If 120 passengers are booked on the flight, in how many ways can the airline choose the seven passengers to be bumped?

$_{120}C_7 = \boxed{5.949 \times 10^{10}}$

4. A quiz has ten true-false questions and five multiple-choice questions with four choices for each. In how many ways can this test be completed?

$2^{10} \cdot 4^5 = \boxed{1,048,576}$

5. If you must answer only eight of ten questions on a test, how many ways do you have of choosing the questions you will omit?

$_{10}C_2 = \boxed{45}$

6. Michael rolls a die seven times. Find the probability of rolling:

a. Exactly four 6's.

$\binom{7}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^3 \approx \boxed{0.0156}$

b. No 6's.

$\binom{7}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^7 = \boxed{0.2791}$

c. No more than three 6's.

$P(0) + P(1) + P(2) + P(3) = \boxed{0.982}$

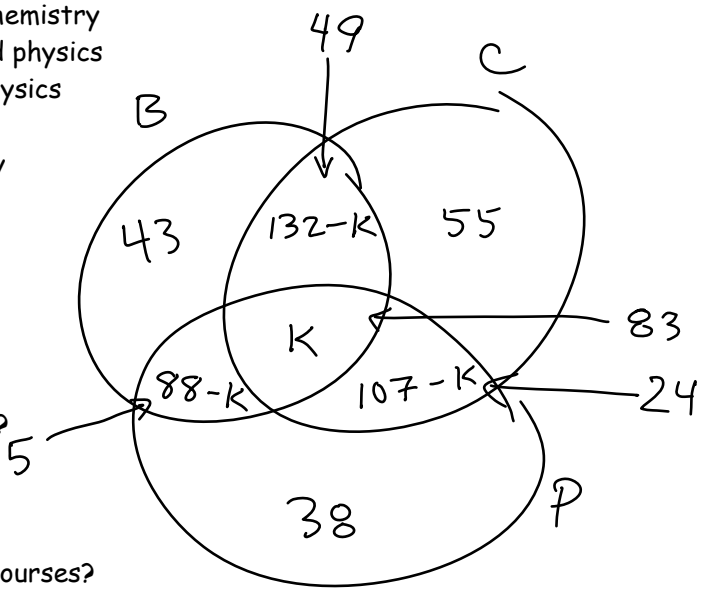
(or use BinomCDF(7, 1/6, 3))

7. Find the number of permutations of the letters of the word "OAKBROOK."

$$\frac{8!}{3!2!} = \boxed{3360}$$

8. All students at Hinsdale Central must take at least one of the following science courses: Biology, Chemistry, and Physics. In order to project the future enrollment in these courses, the principal sent questionnaires to 297 junior high school students and got these results:

132 intend to take biology and chemistry
107 intend to take chemistry and physics
88 intend to take biology and physics
43 intend to take only biology
55 intend to take only chemistry
38 intend to take only physics



a. Make a Venn diagram illustrating the relationships.

b. How many students intend to take biology or physics?

$$297 - 55 = \boxed{242}$$

c. How many students intend to take all three science courses?

$$43 + 55 + 38 + (132-K) + (88-K) + (107-K) + K = 297$$

$$463 - 2K = 297 \Rightarrow 2K = 166 \Rightarrow K = \boxed{83}$$

9. A shelf has ten books: two mysteries, four sci-fi novels, and (Mrs. Tyler's favorite) four math textbooks!

a. Find the number of possible arrangements keeping the types of books together.

$$3!\,2!\,4!\,4! = \boxed{6912}$$

b. If you select one book, what is the probability that it is a math textbook?

$$\frac{4}{10} = \boxed{0.4}$$

c. If you select two books, what is the probability that they are both the same type?

$$\frac{{}_2C_2 + {}_4C_2 + {}_4C_2}{{}_{10}C_2} = \boxed{\frac{13}{45} \text{ or } 0.2\overline{8}}$$

d. If you select two books, what is the probability that neither is a math text?

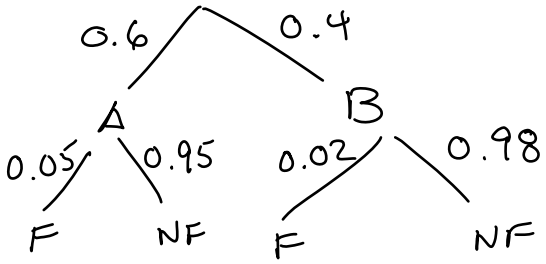
$$\frac{{}_6C_2}{{}_{10}C_2} = \boxed{\frac{1}{3} \text{ or } 0.\overline{3}}$$

e. If you select two books, what is the probability that they are not both math texts?

$$1 - \frac{{}_4C_2}{{}_{10}C_2} = \boxed{\frac{13}{15} \text{ or } 0.8\overline{6}}$$

10. Machine A produces 60% of the ball bearings manufactured by a factory and Machine B produces the rest. Five percent of Machine A's bearings fail to have the required precision, and two percent of Machine B's bearings fail.

a. Draw a tree diagram and label all the probabilities.



b. What percent of the bearing fail to have the required precision?

$(0.6)(0.05) + (0.4)(0.02) = \boxed{0.038}$

c. If a bearing is inspected and fails to have the required precision, what is the probability that it was produced by Machine A?

$\frac{0.03}{0.038} = \boxed{0.7895}$

11. From a well-shuffled deck of 52 cards, two cards are dealt. Find the probability of getting:

a. two face cards

$\frac{{}_{12}C_2}{{}_{52}C_2} = \boxed{.0498}$

b. two tens

$\frac{{}_4C_2}{{}_{52}C_2} = \boxed{0.004525}$

c. two diamonds

$\frac{{}_{13}C_2}{{}_{52}C_2} = \boxed{0.0588}$

d. two face cards or two diamonds

$\frac{{}_{12}C_2 + {}_{13}C_2 - {}_3C_2}{{}_{52}C_2} = \boxed{0.1063}$

e. two face cards or two tens

$\frac{{}_{12}C_2 + {}_4C_2}{{}_{52}C_2} = \boxed{0.0543}$

f. one face card and one ten

$\frac{({}_{12}C_1)({}_4C_1)}{{}_{52}C_2} = \boxed{0.0362}$

12. Of the 1260 households in a small town, 632 have dogs, 568 have cats, and 114 have both types of pet. If a household is chosen at random, what is the probability that the household has a dog or a cat?

D 518 114 454 C

$\frac{1086}{1260} = \frac{181}{210} = \boxed{0.8619}$

13. A box contains 3 red and 5 green holiday pencils. One lucky student will get to randomly select two pencils without replacement. Find the probability that:

a. both pencils are the same color.

$\frac{{}_3C_2 + {}_5C_2}{{}_8C_2} = \boxed{0.4643}$

b. one pencil is red and the other is green.

$\frac{({}_3C_1)({}_5C_1)}{{}_8C_2} = \boxed{0.5357}$

14. "Four out of five dentists recommend Trident to their patients who chew gum." If twelve dentists are asked whether they recommend Trident, what is the probability that:

a. five agree

$\binom{12}{5}\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)^7$

$= 0.003322$

b. eight agree

$\binom{12}{8}\left(\frac{4}{5}\right)^8\left(\frac{1}{5}\right)^4$

$= 0.1329$

c. none agree

$\binom{12}{0}\left(\frac{4}{5}\right)^0\left(\frac{1}{5}\right)^{12}$

$= 4.096 \times 10^{-9}$

d. no more than nine agree

$P(0) + P(1) + \ldots + P(9)$

or $1 - [P(10) + P(11) + P(12)]$

or $\text{BinomCDF}\left(12, \frac{4}{5}, 9\right)$

$= 0.44165$

15. Find the terms whose coefficient in the binomial expansion of $(a + b)^{11}$ is $\binom{11}{4}$.

$\binom{11}{4} a^7 b^4 = 330 a^7 b^4$

($330 a^4 b^7$ would be marginally acceptable)

16. Find the sixth term in the expansion of $(3x - 1)^9$.

$\binom{9}{5}(3x)^4(-1)^5 = -10206x^4$

17. If the fourth and the fourteenth terms in the expansion of $(c + d)^n$ have equal coefficients, find the eleventh term (in terms of c and d).

$\binom{n}{3} = \binom{n}{13}$

$n = 16$

11th term $\rightarrow \binom{16}{10}(c)^{16-10} d^{10}$

$= 8008 c^6 d^{10}$