Precalculus Honors
9.3, 9.4 Review Warmups

1. Consider the sequence: 38, 31, 24, 17, ...
   a) Write a recursive formula for the sequence

$a_1 = 38$
$a_n = a_{n-1} - 7 \quad \text{for} \quad n > 1$

   b) Write an explicit formula for the sequence

$a_n = 38 + (-7)(n-1)$ or $a_n = 45 - 7n$

   c) Which term in the sequence would be -396?

$45 - 7n = -396$
$-7n = -441$
$n = 63$

2. Does the series $2 - 2 + 3 - 3 + 4 - 4 + 5 - 5$ ... converge or diverge? Justify your answer!

diverges.

$S_1 = 2$
$S_2 = 0$
$S_3 = 3$
$S_4 = 0$
$S_5 = 4$
$S_6 = 0$
$\vdots$

The sequence of partial sums does not converge.

3. Evaluate $\sum_{k=1}^{\infty} (20)\left(\frac{-1}{4}\right)^k$

$a_1 = -5$
$r = -\frac{1}{4}$

$S_\infty = \frac{-5}{1-(-1/4)} = \frac{-5}{5/4} = \frac{-20}{5} = -4$

4. Find A_6 for the following sequence:

$A_1 = 8$
$A_n = 20 - 2A_{n-1} \text{ for } n \geq 2$

$A_1 = 8$
$A_2 = 20 - 2(8) = 4$
$A_3 = 20 - 2(4) = 12$
$A_4 = 20 - 2(12) = -4$
$A_5 = 20 - 2(-4) = 28$
$A_6 = 20 - 2(28) = -36$

Honors Precalculus
9.3-9.4 Review with some tough questions

Name Key

1. How many terms -10, -7, -4, …must be added to give a sum of 200?

$200 = \frac{n}{2}\left(2(-10) + 3(n-1)\right)$

$200 = \frac{n}{2}(-23 + 3n)$ $\quad 400 = -23n + 3n^2$

$3n^2 - 23n - 400 = 0$ (graphed) $\quad n = 16$

2. If $t_4 = \frac{1}{2}$ *and* $t_9 = \frac{1}{64}$, find the sum of the first 12 terms of the geometric series.

$r^5 = \frac{1}{32}$

$r = 1/2$

$a_1 = 4$

$S_{12} = 4\left(\frac{1-(.5)^{12}}{1-.5}\right) = \frac{4095}{512} \approx 7.998$

3. Find a_7, if $r = \frac{1}{2}$, n=7, $S_n = \frac{381}{4}$

$\frac{381}{4} = a_1\left(\frac{1-.5^7}{1-.5}\right)$

$\frac{381}{4} = \frac{127}{64}a_1$

$a_1 = 48$

$a_n = 48\left(\frac{1}{2}\right)^{n-1}$

$a_7 = 48\left(\frac{1}{2}\right)^6$

$a_7 = 0.75$

4. Find the sum of the geometric series, using formulas to find what is needed.

$6561 - 2187 + 729 - 243 + \ldots + \frac{1}{729}$

$\frac{1}{729} = 6561\left(-\frac{1}{3}\right)^{n-1}$

$\frac{1}{4{,}782{,}969} = \left(-\frac{1}{3}\right)^{n-1}$

$\left(-\frac{1}{3}\right)^{-14} = (-3)^{-n+1}$

$-n+1 = -14$

$n = 15$

$S_{15} = 6561\left(\frac{1-(-\frac{1}{3})}{1-(-\frac{1}{3})}\right)$

$S_{15} = 4920.75$

5. Find the range of values of x for which the following geometric series converges:

$1 + 3(4-x) + 9(4-x)^2 + 27(4-x)^3 + \ldots$

$\frac{3(4-x)}{1} = r$ $\quad r = 3(4-x)$ $\quad |r| < 1$ $\therefore$ $-1 < 3(4-x) < 1$

$-1 < 12 - 3x < 1$

$-13 < -3x < -11$

$13\frac{1}{3} > x > 11/13$

HPC
9.3, 9.4 More Practice

Name:

1. Find a_{60} when $a_1 = 35$, $d = -3$.

$a_{60} = a_1 + d(n-1)$
$= 35 + -3(59)$
$= \boxed{-142}$

2. Write the formula for the general term. Then find the 20th term. $a_n = a_{n-1} + 3$, $a_1 = 4$

$a_{n-1} = 4 - 3 = 1$ $\quad d = 3$ $\quad a_1 = 4$
$\boxed{a_n = 1 + 3n}$
$a_{20} = 1 + 3(20) = \boxed{61}$

3. $\sum_{i=1}^{10} 5 \cdot 2^i$

$a_1 = 10$
$r = 2$
$= 10\left(\frac{1-2^{10}}{1-2}\right) = \boxed{10,230}$

4. $\sum_{i=1}^{\infty} 8(-0.3)^{i-1}$

$a_1 = 8(-0.3)^0 = 8$
$= \frac{a_1}{1-r} = \frac{8}{1-(-.3)} = \boxed{\frac{80}{13}}$

5. Company A pays \$24,000 yearly with raises of \$1600 per year. Company B pays \$28,000 yearly with raises of \$1000 per year. Which company will pay more in year 10? How much more?

Company A: $d = 1600$
$a_{10} = 24,000 + 1600(9)$
$= \$38,400$

Company B: $d = 1000$
$a_{10} = 28,000 + 1000(9)$
$= 37,000$

Comp. A will pay \$1400 more in year 10.

6. A professional baseball player signs a contract with a beginning salary of \$3,000,000 for the first year and an annual increase of 4% per year beginning in the second year. What is the athlete's salary for year 7 of the contract? Geometric

$a_n = a_1 \cdot r^{n-1}$
$a_7 = 3,000,000(1.04)^6 = \boxed{\$3,795,957.06}$

7. A section in a stadium has 20 seats in the first row, 23 seats in the second row, increasing by 3 seats each row for a total of 38 rows. How many seats are in this section of the stadium? Arith.

$a_1 = 20$
$d = 3$
$n = 38$

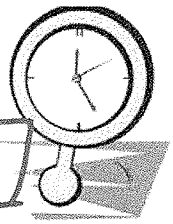
$\frac{n}{2}(2a_1 + d(n-1))$
$\frac{38}{2}[2(20) + 3(37)] = \boxed{2869 \text{ seats}}$

8. A pendulum swings through an arc of 20 inches. On each successive swing, the length of the arc is 90% of the previous length. After 10 swings, what is the total length of the distance the pendulum has swung?

1st swing: 20
2nd swing: .9(20)
3rd : $.9^2(20)$

Total: $\sum_{i=1}^{10} .9^{i-1}(20)$

$= 20\left[\frac{1-.9^{10}}{1-.9}\right] \approx$ 130.26 in



9. Determine whether the following statement makes sense or not. Explain. *"I modeled California's population growth with a geometric sequence, so my model is an exponential function whose domain is the set of natural numbers."*

Makes sense.

exp. growth → geom.

Practice!!!! Converge or Diverge (if it converges, please give limit!)

3. 20, 15, 10, 5, ... Diverge

4. $a_n = \frac{(-1)^n}{2n-1}$ Converge to 0

5. $a_n = n \cos \pi$ $\cos \pi = -1$ Diverge

6. $\frac{3n}{n+1}$ Converge to 3

$\lim_{n\to\infty} \frac{3n}{n+1} = \frac{3}{1} = 3$

7. $\frac{5n^2}{n^3+1}$ $\lim_{n\to\infty} \frac{5}{n} =$ Converge to 0

8. $\frac{n^3+2}{n^2+n}$ $\lim_{n\to\infty} \frac{n}{1} = \infty$

Diverges