Precalculus Honors
Practice §§2.5 - 2.7

Name KEY
Period

NO CALCULATORS!!

1. Write the unique polynomial of minimum degree with <u>real coefficients</u> in standard form that meets the following conditions:
Zeros: 2-*i*, -2
f(1) = 18

Since real coeffs and 2-i is a zero → 2+i is also a zero

[similar to p. 215: 5-16, 49, 50]

$$f(x) = a\underbrace{(x-(2-i))(x-(2+i))}(x+2)$$
$$= a\left(x^2-2(2)x+(2^2+1^2)\right)(x+2)$$
$$= a(x^2-4x+5)(x+2)$$
$$= a\left(\begin{matrix}x^3-4x^2+5x \\ +2x^2-8x+10\end{matrix}\right)$$
$$f(x) = a(x^3-2x^2-3x+10)$$

$$f(1)=18 \rightarrow 18 = a(1^3-2(1)^2-3(1)+10)$$
$$18 = a(1-2-3+10)$$
$$18 = 6a$$
$$a=3$$
$$\boxed{f(x) = 3x^3-6x^2-9x+30}$$

2. Using the fact that 2+i is a zero of the function $f(x)=3x^4-10x^3+6x^2+14x-5$, find all of the zeros of the function.

[similar to p. 215: 33 - 36]

real coefs + 2+i is a zero → 2-i is also a zero

$(x-(2+i))(x-(2-i))$
(x^2-4x+5) ← combine the two known factors

NOW DIVIDE THIS COMBINED FACTOR OUT

NOW FACTOR WHAT IS LEFT USING QUADRATIC FORMULA OR BASIC TECHNIQUES

$$\begin{array}{r|l} & 3x^2+2x-1 \\ \hline x^2-4x+5 & 3x^4-10x^3+6x^2+14x-5 \\ & -(3x^4-12x^3+15x^2) \\ \hline & 2x^3-9x^2+14x \\ & -(2x^3-8x^2+10x) \\ & -x^2+4x-5 \\ & -(-x^2+4x-5) \\ \hline & 0 \end{array}$$

$$x = \frac{-2 \pm \sqrt{4-4(3)(-1)}}{6}$$
$$= \frac{-2 \pm \sqrt{4+12}}{6}$$
$$= \frac{-2 \pm \sqrt{16}}{6} = \frac{-2 \pm 4}{6} = -1,\ 1/3$$
$$\boxed{-1,\ 1/3,\ 2+i,\ 2-i}$$

3. Solve: $\dfrac{3x}{x+3}+\dfrac{3}{x-2}=\dfrac{15}{x^2+x-6}$

[similar to p. 232: 11-18, 23 - 28]

LCD = $(x+3)(x-2)$

$$\frac{3x}{\cancel{x+3}}\cancel{(x+3)}(x-2) + \frac{3}{\cancel{x-2}}(x+3)\cancel{(x-2)} = \frac{15}{\cancel{(x+3)(x-2)}}\cdot\cancel{(x+3)(x-2)}$$

$$3x(x-2) + 3(x+3) = 15$$
$$3x^2-6x+3x+9 = 15$$
$$3x^2-3x-6=0$$
$$x^2-x-2=0$$

$$(x-2)(x+1)=0$$
$\cancel{x=}2$, $\boxed{x=-1}$
extr.

4. Describe how the graph of $g(x)=\frac{4x-3}{2x+1}$ is obtained by transforming the graph of $f(x)=\frac{1}{x}$

[similar to p. 225: 5-10]

LONG DIVISION:

$$\begin{array}{r|l} & \;\;2 \\ 2x+1 & 4x-3 \\ & -(4x+2) \\ \hline & \;\;-5 \end{array}$$

$$\frac{4x-3}{2x+1} = \boxed{2+\frac{-5}{2x+1}} = \boxed{2+-5\left(\frac{1}{2x+1}\right)}$$

SHIFT LEFT 1
HORIZ SHRINK BAFO ½
REFLECT ACROSS X-AXIS
VERTICAL STRETCH BAFO 5
SHIFT UP 2

5. Identify the characteristics of

$$f(x)=\frac{x^3+4x^2-5x}{x^2-2x-3} = \frac{x(x^2+4x-5)}{(x-3)(x+1)} = \frac{x(x+5)(x-1)}{(x-3)(x+1)}$$

Vertical asymptotes: $\boxed{\begin{matrix} x=3 \\ x=-1 \end{matrix}}$

Horizontal asymptotes:

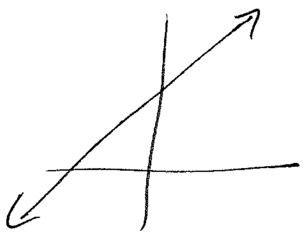
$\boxed{\text{NONE}}$ (NUM DEGREE > DENOM DEGREE)

End Behavior asymptote:

$$\begin{array}{r|l} & \qquad\quad x+6 \\ x^2-2x-3 & x^3+4x^2-5x \\ & -(x^3-2x^2-3x) \\ \hline & \qquad 6x^2-2x \\ & \quad -(6x^2-12x-18) \\ \hline & \qquad\; 10x+18 \end{array}$$

$\boxed{y=x+6}$

EBA



x-intercepts:

$\boxed{\{0,-5,1\}}$

y-intercepts:

$f(0)=\boxed{0}$

Describe the end behavior using limit statements:

$\boxed{\lim_{x\to-\infty} f(x)=-\infty}$ $\boxed{\lim_{x\to\infty} f(x)=\infty}$ (you can find these easily from EBA)

Describe the behavior on either side of the vertical asymptotes using limit statements:

$$\frac{x(x+5)(x-1)}{(x-3)(x+1)}$$

$\boxed{\frac{-+-}{--}}$ $\boxed{\lim_{x\to-1^-} f(x)=\infty}$

$\boxed{\frac{-+-}{-+}}$ $\boxed{\lim_{x\to-1^+} f(x)=-\infty}$

$\boxed{\lim_{x\to3^-} f(x)=-\infty}$ $\boxed{\frac{+++}{-+}}$

[similar to p. 226: 37-52]

$\boxed{\lim_{x\to3^+} f(x)=\infty}$ $\boxed{\frac{+++}{++}}$