47. $2x^2 + 7x - 15 = 10$ or $2x^2 + 7x - 15 = -10$

$2x^2 + 7x - 25 = 0$ $\quad$ $2x^2 + 7x - 5 = 0$

The graph of $y = 2x^2 + 7x - 25$ appears to be zero for $x \approx -5.69$ and $x \approx 2.19$

The graph of $y = 2x^2 + 7x - 5$ appears to be zero for $x \approx -4.11$ and $x \approx 0.61$

Now look at the graphs of $y = |2x^2 + 7x - 15|$ and $y = 10$. The graph of $y = |2x^2 + 7x - 15|$ lies below the graph of $y = 10$ when $-5.69 < x < -4.11$ and when $0.61 < x < 2.19$. Hence $(-5.69, -4.11) \cup (0.61, 2.19)$ is the approximate solution.

48. $2x^2 + 3x - 20 = 10$ or $2x^2 + 3x - 20 = -10$

$2x^2 + 3x - 30 = 0$ $\quad$ $2x^2 + 3x - 10 = 0$

The graph of $y = 2x^2 + 3x - 30$ appears to be zero for $x \approx -4.69$ and $x \approx 3.19$

The graph of $y = 2x^2 + 3x - 10$ appears to be zero for $x \approx -3.11$ and $x \approx 1.61$

Now look at the graphs of $y = |2x^2 + 3x - 20|$ and $y = 10$. The graph of $y = |2x^2 + 7x - 20|$ lies above the graph of $y = 10$ when $x < -4.69$, $-3.11 < x < 1.61$, and $x > 3.19$. Hence $(-\infty, -4.69] \cup [-3.11, 1.61] \cup [3.19, \infty)$ is the (approximate) solution.

Chapter P Review

1. Endpoints 0 and 5; bounded

2. Endpoint 2; unbounded

3. $2(x^2 - x) = 2x^2 - 2x$

4. $2x^3 + 4x^2 = 2x^2 \cdot x + 2x^2 \cdot 2 = 2x^2(x + 2)$

5. $\dfrac{(uv^2)^3}{v^2u^3} = \dfrac{u^3v^6}{u^3v^2} = v^4$

6. $(3x^2y^3)^{-2} = \dfrac{1}{(3x^2y^3)^2} = \dfrac{1}{3^2(x^2)^2(y^3)^2} = \dfrac{1}{9x^4y^6}$

7. 3.68×10^9

8. 7×10^{-6}

9. 5,000,000,000

10. 0.000 000 000 000 000 000 000 000 000 910 94
(27 zeros between the decimal point and the first 9)

11. **(a)** 5.0711×10^{10}

(b) 4.63×10^9

(c) 5.0×10^8

(d) 3.995×10^9

(e) 1.4497×10^{10}

12. $-0.\overline{45}$ (repeating)

13. **(a)** Distance: $|14 - (-5)| = |19| = 19$

(b) Midpoint: $\dfrac{-5 + 14}{2} = \dfrac{9}{2} = 4.5$

14. **(a)** Distance:

$\sqrt{[5 - (-4)]^2 + (-1 - 3)^2} = \sqrt{9^2 + (-4)^2}$
$= \sqrt{81 + 16} = \sqrt{97} \approx 9.85$

(b) Midpoint:

$\left(\dfrac{-4 + 5}{2}, \dfrac{3 + (-1)}{2}\right) = \left(\dfrac{1}{2}, \dfrac{2}{2}\right) = \left(\dfrac{1}{2}, 1\right)$

15. The three side lengths (distances between pairs of points) are

$\sqrt{[3 - (-2)]^2 + (11 - 1)^2} = \sqrt{5^2 + 10^2}$
$= \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$

$\sqrt{(7 - 3)^2 + (9 - 11)^2} = \sqrt{4^2 + (-2)^2}$
$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$

$\sqrt{[7 - (-2)]^2 + (9 - 1)^2} = \sqrt{9^2 + 8^2}$
$= \sqrt{81 + 64} = \sqrt{145}.$

Since $(2\sqrt{5})^2 + (5\sqrt{5})^2 = 20 + 125 = 145 = (\sqrt{145})^2$—the sum of the squares of the two shorter side lengths equals the square of the long side length—the points determine a right triangle.

16. The three side lengths (distances between pairs of points) are

$\sqrt{(4 - 0)^2 + (1 - 1)^2} = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$

$\sqrt{(2 - 0)^2 + [(1 - 2\sqrt{3}) - 1]^2} = \sqrt{2^2 + (-2\sqrt{3})^2}$
$= \sqrt{4 + 12} = \sqrt{16} = 4$

$\sqrt{(4 - 2)^2 + [1 - (1 - 2\sqrt{3})]^2} = \sqrt{2^2 + (2\sqrt{3})^2}$
$= \sqrt{4 + 12} = \sqrt{16} = 4.$

Since all three sides have the same length, the figure is an equilateral triangle.

17. $(x - 0)^2 + (y - 0)^2 = 2^2$, or $x^2 + y^2 = 4$

18. $(x - 5)^2 + [y - (-3)]^2 = 4^2$, or
$(x - 5)^2 + (y + 3)^2 = 16$

19. $[x - (-5)]^2 + [y - (-4)]^2 = 3^2$, so the center is $(-5, -4)$ and the radius is 3.

20. $(x - 0)^2 + (y - 0)^2 = 1^2$, so the center is $(0, 0)$ and the radius is 1.

21. **(a)** Distance between $(-3, 2)$ and $(-1, -2)$:

$\sqrt{(-2 - 2)^2 + [-1 - (-3)]^2} = \sqrt{(-4)^2 + (2)^2}$
$= \sqrt{16 + 4} = \sqrt{20} \approx 4.47$

Distance between $(-3, 2)$ and $(5, 6)$:

$\sqrt{(6 - 2)^2 + [5 - (-3)]^2} = \sqrt{4^2 + 8^2}$
$= \sqrt{16 + 64} = \sqrt{80} \approx 8.94$

Distance between $(5, 6)$ and $(-1, -2)$:

$\sqrt{(-2 - 6)^2 + (-1 - 5)^2} = \sqrt{(-8)^2 + (-6)^2}$
$= \sqrt{64 + 36} = \sqrt{100} = 10$

(b) $(\sqrt{20})^2 + (\sqrt{80})^2 = 20 + 80 = 100 = 10^2$, so the Pythagorean Theorem guarantees the triangle is a right triangle.

22. $|z - (-3)| \le 1$, or $|z + 3| \le 1$

23. $\dfrac{-1 + a}{2} = 3$ and $\dfrac{1 + b}{2} = 5$

$-1 + a = 6$ $\quad$ $1 + b = 10$

$a = 7$ $\quad$ $b = 9$

24. $m = \dfrac{-5 + 2}{4 + 1} = -\dfrac{3}{5}$

25. $y + 1 = -\dfrac{2}{3}(x - 2)$

26. The slope is $m = -\frac{9}{7} = -\frac{A}{B}$, so we can choose $A = 9$ and $B = 7$. Since $x = -5$, $y = 4$ solves $9x + 7y + C = 0$, C must equal 17: $9x + 7y + 17 = 0$. Note that the coefficients can be multiplied by any non-zero number, e.g., another answer would be $18x + 14y + 34 = 0$.

27. Beginning with point-slope form: $y + 2 = \frac{4}{5}(x - 3)$, so $y = \frac{4}{5}x - 4.4$.

28. $m = \frac{2 + 4}{3 + 1} = \frac{3}{2}$, so in point-slope form, $y + 4 = \frac{3}{2}(x + 1)$, and therefore $y = \frac{3}{2}x - \frac{5}{2}$.

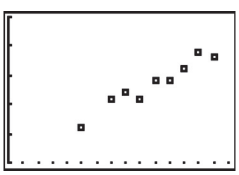
29. $y = 4$

30. Solve for y: $y = \frac{3}{4}x - \frac{7}{4}$.

31. The slope of the given line is the same as the line we want: $m = -\frac{2}{5}$, so $y + 3 = -\frac{2}{5}(x - 2)$, and therefore $y = -\frac{2}{5}x - \frac{11}{5}$.

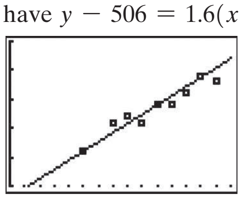
32. The slope of the given line is $-\frac{2}{5}$, so the slope of the line we seek is $m = \frac{5}{2}$. Then $y + 3 = \frac{5}{2}(x - 2)$, and therefore $y = \frac{5}{2}x - 8$.

33. **(a)**



[0, 15] by [500, 525]

(b) Slope of the line between the points (5, 506) and (10, 514) is $m = \frac{514 - 506}{10 - 5} = \frac{8}{5} = 1.6$.
Using the *point-slope form* equation for the line, we have $y - 506 = 1.6(x - 5)$, so $y = 1.6x + 498$.



[0, 15] by [500, 525]

(c) The year 1996 is represented by $x = 6$. Using $y = 1.6x + 498$ and $x = 6$, we estimate the average SAT math score in 1996 to be 507.6, which is very close to the actual value 508.

(d) The year 2006 is represented by $x = 16$. Using $y = 1.6x + 498$ and $x = 16$, we predict the average SAT math score in 2006 will be 524.

34. **(a)** $4x - 3y = -33$, or $y = \frac{4}{3}x + 11$

(b) $3x + 4y = -6$, or $y = -\frac{3}{4}x - \frac{3}{2}$

35. $m = \frac{25}{10} = \frac{5}{2} = 2.5$

36. Both graphs look the same, but the graph on the left has slope $\frac{2}{3}$—less than the slope of the one on the right, which is $\frac{12}{15} = \frac{4}{5}$. The different horizontal and vertical scales for the two windows make it difficult to judge by looking at the graphs.

37.
$$3x - 4 = 6x + 5$$
$$-3x = 9$$
$$x = -3$$

38.
$$\frac{x-2}{3} + \frac{x+5}{2} = \frac{1}{3}$$
$$2(x - 2) + 3(x + 5) = 2$$
$$2x - 4 + 3x + 15 = 2$$
$$5x + 11 = 2$$
$$5x = -9$$
$$x = -\frac{9}{5}$$

39.
$$2(5 - 2y) - 3(1 - y) = y + 1$$
$$10 - 4y - 3 + 3y = y + 1$$
$$7 - y = y + 1$$
$$-2y = -6$$
$$y = 3$$

40.
$$3(3x - 1)^2 = 21$$
$$(3x - 1)^2 = 7$$
$$3x - 1 = \pm\sqrt{7}$$
$$3x - 1 = -\sqrt{7} \quad \text{or} \quad 3x - 1 = \sqrt{7}$$
$$x = \frac{1}{3} - \frac{\sqrt{7}}{3} \approx -0.55 \quad x = \frac{1}{3} + \frac{\sqrt{7}}{3} \approx 1.22$$

41.
$$x^2 - 4x - 3 = 0$$
$$x^2 - 4x = 3$$
$$x^2 - 4x + (2)^2 = 3 + (2)^2$$
$$(x - 2)^2 = 7$$
$$x - 2 = \pm\sqrt{7}$$
$$x - 2 = -\sqrt{7} \quad \text{or} \quad x - 2 = \sqrt{7}$$
$$x = 2 - \sqrt{7} \approx -0.65 \quad x = 2 + \sqrt{7} \approx 4.65$$

42. $16x^2 - 24x + 7 = 0$
Using the quadratic formula:
$$x = \frac{24 \pm \sqrt{24^2 - 4(16)(7)}}{2(16)}$$
$$= \frac{24 \pm \sqrt{128}}{32} = \frac{3}{4} \pm \frac{\sqrt{2}}{4}$$
$$x = \frac{3}{4} - \frac{\sqrt{2}}{4} \approx 0.40 \quad \text{or} \quad x = \frac{3}{4} + \frac{\sqrt{2}}{4} \approx 1.10$$

43.
$$6x^2 + 7x = 3$$
$$6x^2 + 7x - 3 = 0$$
$$(3x - 1)(2x + 3) = 0$$
$$3x - 1 = 0 \quad \text{or} \quad 2x + 3 = 0$$
$$x = \frac{1}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

44. $2x^2 + 8x = 0$
$2x(x + 4) = 0$
$2x = 0$ or $x + 4 = 0$
$x = 0$ or $x = -4$

45. $x(2x + 5) = 4(x + 7)$
$2x^2 + 5x = 4x + 28$
$2x^2 + x - 28 = 0$
$(2x - 7)(x + 4) = 0$
$2x - 7 = 0$ or $x + 4 = 0$
$x = \frac{7}{2}$ or $x = -4$

46. $4x + 1 = 3$ or $4x + 1 = -3$
$4x = 2$ or $4x = -4$
$x = \frac{1}{2}$ or $x = -1$

47. $4x^2 - 20x + 25 = 0$
$(2x - 5)(2x - 5) = 0$
$(2x - 5)^2 = 0$
$2x - 5 = 0$
$x = \frac{5}{2}$

48. $-9x^2 + 12x - 4 = 0$
$9x^2 - 12x + 4 = 0$
$(3x - 2)(3x - 2) = 0$
$(3x - 2)^2 = 0$
$3x - 2 = 0$
$x = \frac{2}{3}$

49. $x^2 = 3x$
$x^2 - 3x = 0$
$x(x - 3) = 0$
$x = 0$ or $x - 3 = 0$
$x = 0$ or $x = 3$

50. Solving $4x^2 - 4x + 2 = 0$ by using the quadratic formula with $a = 4$, $b = -4$, and $c = 2$ gives

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(2)}}{2(4)} = \frac{4 \pm \sqrt{-16}}{8}$$
$$= \frac{4 \pm 4i}{8} = \frac{1}{2} \pm \frac{1}{2}i$$

51. Solving $x^2 - 6x + 13 = 0$ by using the quadratic formula with $a = 1$, $b = -6$, and $c = 13$ gives

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)} = \frac{6 \pm \sqrt{-16}}{2}$$
$$= \frac{6 \pm 4i}{2} = 3 \pm 2i$$

52. Solving $x^2 - 2x + 4 = 0$ by using the quadratic formula with $a = 1$, $b = -2$, and $c = 4$ gives

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} = \frac{2 \pm \sqrt{-12}}{2}$$
$$= \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm \sqrt{3}i$$

53. $2x^2 - 3x - 1 = 0$
$$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$$
$$x^2 - \frac{3}{2}x + \left(-\frac{3}{4}\right)^2 = \frac{1}{2} + \left(-\frac{3}{4}\right)^2$$
$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$
$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$
$$x = \frac{3}{4} \pm \frac{\sqrt{17}}{4}$$
$$x = \frac{3}{4} - \frac{\sqrt{17}}{4} \approx -0.28 \quad \text{or} \quad x = \frac{3}{4} + \frac{\sqrt{17}}{4} \approx 1.78$$

54. $3x^2 + 4x - 1 = 0$
$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-1)}}{2(3)}$$
$$= \frac{-4 \pm \sqrt{28}}{6} = -\frac{2}{3} \pm \frac{\sqrt{7}}{3}$$
$$x = -\frac{2}{3} - \frac{\sqrt{7}}{3} \approx -1.55 \text{ or } x = -\frac{2}{3} + \frac{\sqrt{7}}{3} \approx 0.22$$

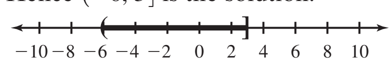
55. The graph of $y = 3x^3 - 19x^2 - 14x$ is zero for $x = 0$, $x = -\frac{2}{3}$, and $x = 7$.

56. The graph of $y = x^3 + 2x^2 - 4x - 8$ is zero for $x = -2$, and $x = 2$.

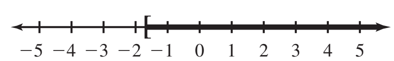
57. The graph of $y = x^3 - 2x^2 - 2$ is zero for $x \approx 2.36$.

58. The graph of $y = |2x - 1| - 4 + x^2$ is zero for $x = -1$ and for $x \approx 1.45$.

59. $-2 < x + 4 \le 7$
$-6 < x \le 3$
Hence $(-6, 3]$ is the solution.



60. $5x + 1 \ge 2x - 4$
$3x \ge -5$
$x \ge -\frac{5}{3}$
Hence $\left[-\frac{5}{3}, \infty\right)$ is the solution.



61. $\frac{3x - 5}{4} \le -1$
$3x - 5 \le -4$
$3x \le 1$
$x \le \frac{1}{3}$
Hence $\left(-\infty, \frac{1}{3}\right]$ is the solution.

62. $-7 < 2x - 5 < 7$
$-2 < 2x < 12$
$-1 < x < 6$
Hence $(-1, 6)$ is the solution.

63. $3x + 4 \geq 2$ or $3x + 4 \leq -2$

$3x \geq -2$ or $3x \leq -6$

$x \geq -\frac{2}{3}$ or $x \leq -2$

Hence $(-\infty, -2] \cup \left[-\frac{2}{3}, \infty\right)$ is the solution.

64. $4x^2 + 3x - 10 = 0$

$(4x - 5)(x + 2) = 0$

$4x - 5 = 0$ or $x + 2 = 0$

$x = \frac{5}{4}$ or $x = -2$

The graph of $y = 4x^2 + 3x - 10$ lies above the x-axis for $x < -2$ and for $x > \frac{5}{4}$. Hence $(-\infty, -2) \cup \left(\frac{5}{4}, \infty\right)$ is the solution.

65. The graph of $y = 2x^2 - 2x - 1$ is zero for $x \approx -0.37$ and $x \approx 1.37$, and lies above the x-axis for $x < -0.37$ and for $x > 1.37$. Hence $(-\infty, -0.37) \cup (1.37, \infty)$ is the approximate solution.

66. The graph of $y = 9x^2 - 12x - 1$ is zero for $x \approx -0.08$, and $x \approx 1.41$, and lies below the x-axis for $-0.08 < x < 1.41$. Hence $[-0.08, 1.41]$ is the approximate solution.

67. $x^3 - 9x \leq 3$ is equivalent to $x^3 - 9x - 3 \leq 0$. The graph of $y = x^3 - 9x - 3$ is zero for $x \approx -2.82$, $x \approx -0.34$, and $x \approx 3.15$, and lies below the x-axis for $x < -2.82$ and for $-0.34 < x < 3.15$. Hence the approximate solution is $(-\infty, -2.82] \cup [-0.34, 3.15]$.

68. The graph of $y = 4x^3 - 9x + 2$ is zero for $x \approx -1.60$, $x \approx 0.23$, and $x \approx 1.37$, and lies above the x-axis for $-1.60 < x < 0.23$ and for $x > 1.37$. Hence the approximate solution is $(-1.60, 0.23) \cup (1.37, \infty)$.

69. $\frac{x + 7}{5} > 2$ or $\frac{x + 7}{5} < -2$

$x + 7 > 10$ or $x + 7 < -10$

$x > 3$ or $x < -17$

Hence $(-\infty, -17) \cup (3, \infty)$ is the solution.

70. $2x^2 + 3x - 35 = 0$

$(2x - 7)(x + 5) = 0$

$2x - 7 = 0$ or $x + 5 = 0$

$x = \frac{7}{2}$ or $x = -5$

The graph of $y = 2x^2 + 3x - 35$ lies below the x-axis for $-5 < x < \frac{7}{2}$. Hence $\left(-5, \frac{7}{2}\right)$ is the solution.

71. $4x^2 + 12x + 9 = 0$

$(2x + 3)(2x + 3) = 0$

$(2x + 3)^2 = 0$

$2x + 3 = 0$

$x = -\frac{3}{2}$

The graph of $y = 4x^2 + 12x + 9$ lies entirely above the x-axis except for $x = -\frac{3}{2}$. Hence all real numbers satisfy the inequality. So $(-\infty, \infty)$ is the solution.

72. $x^2 - 6x + 9 = 0$

$(x - 3)(x - 3) = 0$

$(x - 3)^2 = 0$

$x - 3 = 0$

$x = 3$

The graph of $y = x^2 - 6x + 9$ lies entirely above the x-axis except for $x = 3$. Hence no real number satisfies the inequality. There is no solution.

73. $(3 - 2i) + (-2 + 5i) = (3 - 2) + (-2 + 5)i$
$= 1 + 3i$

74. $(5 - 7i) - (3 - 2i) = (5 - 3) + (-7 + 2)i$
$= 2 - 5i$

75. $(1 + 2i)(3 - 2i) = 3 - 2i + 6i - 4i^2$
$= 3 + 4i + 4$
$= 7 + 4i$

76. $(1 + i)^3 = ((1 + i)(1 + i))(1 + i)$
$= (1 + 2i + i^2)(1 + i) = 2i(1 + i)$
$= 2i + 2i^2 = -2 + 2i$

77. $(1 + 2i)^2(1 - 2i)^2 = (1 + 4i + 4i^2)(1 - 4i + 4i^2)$
$= (-3 + 4i)(-3 - 4i)$
$= 9 - 12i + 12i - 16i^2 = 25$

78. $i^{29} = i^{28}i = (i^2)^{14}\, i = (-1)^{14}\, i = i$

79. $\sqrt{-16} = \sqrt{(16)(-1)} = 4\sqrt{-1} = 4i$

80. $\frac{2 + 3i}{1 - 5i} = \frac{2 + 3i}{1 - 5i} \cdot \frac{1 + 5i}{1 + 5i} = \frac{2 + 10i + 3i + 15i^2}{1 + 5i - 5i - 25i^2}$

$= \frac{-13 + 13i}{26} = -\frac{1}{2} + \frac{1}{2}i$

81. $s = -16t^2 + 320t$

(a) $-16t^2 + 320t = 1538$

$-16t^2 + 320t - 1538 = 0$

The graph of $s = -16t^2 + 320t - 1538$ is zero at

$$t = \frac{-320 \pm \sqrt{320^2 - 4(-16)(-1538)}}{2(-16)}$$

$$= \frac{-320 \pm \sqrt{3968}}{-32} = \frac{40 \pm \sqrt{62}}{4}.$$

So $t = \frac{40 - \sqrt{62}}{4} \approx 8.03$ sec or

$t = \frac{40 + \sqrt{62}}{4} \approx 11.97.$

The projectile is 1538 ft above ground twice: at $t \approx 8$ sec, on the way up, and at $t \approx 12$ sec, on the way down.

(b) The graph of $s = -16t^2 + 320t$ lies below the graph of $s = 1538$ for $0 < t < 8$ and for $12 < t < 20$ (approximately). Hence the projectile's height will be at most 1538 ft when t is in the interval $(0, 8]$ or $[12, 20)$ (approximately).

(c) The graph of $s = -16t^2 + 320t$ lies above the graph of $s = 1538$ for $8 < t < 12$ (approximately). Hence the projectile's height will be greater than or equal to 1538 when t is in the interval $[8, 12]$ (approximately).

48. Solve $48{,}814.20 = x + 0.12x + 0.03x + 0.004x$. Then $48{,}814.20 = 1.154x$, so $x = 42{,}300$ dollars.

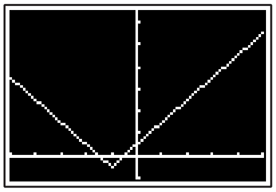
49. **(a)** $y_1 = u(x) = 125{,}000 + 23x$.

(b) $y_2 = s(x) = 125{,}000 + 23x + 8x = 125{,}000 + 31x$.

(c) $y_3 = r_u(x) = 56x$.

(d) $y_4 = R_s(x) = 79x$.

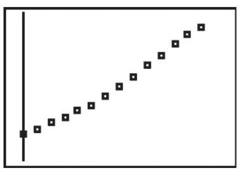
(e)



$[-10, 10]$ by $[-2, 18]$

(f) You should recommend stringing the rackets; fewer strung rackets need to be sold to begin making a profit (since the intersection of y_2 and y_4 occurs for smaller x than the intersection of y_1 and y_3).

50. **(a)**



[–1, 15] by [9, 16]

(b) $y = 0.409x + 9.861$

(c) $r = 0.993$, so the linear model is appropriate.

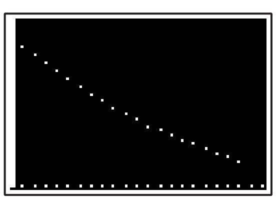
(d) $y = 0.012x^2 + 0.247x + 10.184$

(e) $r^2 = 0.998$, so a quadratic model is appropriate.

(f) The linear prediction is 18.04 and the quadratic prediction is 19.92. Despite the fact that both models look good for the data, the predictions differ by 1.88. One or both of them must be ineffective, as they both cannot be right.

(g) The linear regression and the quadratic regression are very close from $x = 0$ to $x = 13$. The quadratic regression begins to veer away from the linear regression at $x = 13$. Since there are no data points beyond $x = 13$, it is difficult to know which is accurate.

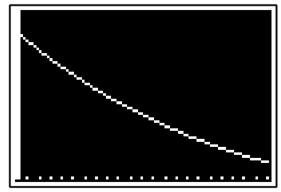
51. **(a)**



$[0, 22]$ by $[100, 200]$

(b) List L3 = {112.3, 106.5, 101.5, 96.6, 92.0, 87.2, 83.1, 79.8, 75.0, 71.7, 68, 64.1, 61.5, 58.5, 55.9, 53.0, 50.8, 47.9, 45.2, 43.2}

(c) The regression equation is $y = 118.07 \times 0.951^x$. It fits the data extremely well.



$[0, 22]$ by $[100, 200]$

52. Answers will vary in **(a)–(e)**, depending on the conditions of the experiment.

(f) Some possible answers: the thickness of the liquid, the darkness of the liquid, the type of cup it is in, the amount of surface exposed to the air, the specific heat of the substance (a technical term that may have been learned in physics), etc.

■ Chapter 1 Review

1. (d)

2. (f)

3. (i)

4. (h)

5. (b)

6. (j)

7. (g)

8. (c)

9. (a)

10. (e)

11. **(a)** All reals **(b)** All reals

12. **(a)** All reals **(b)** All reals

13. **(a)** All reals

(b) $g(x) = x^2 + 2x + 1 = (x + 1)^2$.
At $x = -1$, $g(x) = 0$, the function's minimum.
The range is $[0, \infty)$.

14. **(a)** All reals

(b) $(x - 2)^2 \geq 0$ for all x, so $(x - 2)^2 + 5 \geq 5$ for all x. The range is $[5, \infty)$.

15. **(a)** All reals

(b) $|x| \geq 0$ for all x, so $3|x| \geq 0$ and $3|x| + 8 \geq 8$ for all x. The range is $[8, \infty)$.

16. **(a)** We need $\sqrt{4 - x^2} \geq 0$ for all x, so $4 - x^2 \geq 0$, $4 \geq x^2$, $-2 \leq x \leq 2$. The domain is $[-2, 2]$.

(b) $0 \leq \sqrt{4 - x^2} \leq 2$ for all x, so
$-2 \leq \sqrt{4 - x^2} - 2 \leq 0$ for all x. The range is $[-2, 0]$.

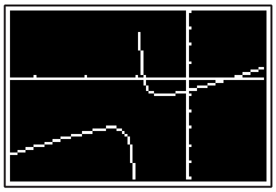
17. **(a)** $f(x) = \dfrac{x}{x^2 - 2x} = \dfrac{x}{x(x - 2)}$. $x \neq 0$ and $x - 2 \neq 0$, $x \neq 2$. The domain is all reals except 0 and 2.

(b) For $x > 2$, $f(x) > 0$ and for $x < 2$, $f(x) < 0$. $f(x)$ does not cross $y = 0$, so the range is all reals except $f(x) = 0$.

18. (a) We need
$\sqrt{9 - x^2} > 0, 9 - x^2 > 0, 9 > x^2, -3 < x < 3.$
The domain is $(-3, 3)$.

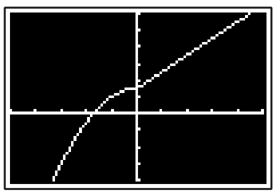
(b) Since $\sqrt{9 - x^2} > 0, \dfrac{1}{\sqrt{9 - x^2}} > 0$. On the domain $(-3, 3), k(0) = \dfrac{1}{3}$, a minimum, while $k(x)$ approaches ∞ when x approaches both -3 and 3, maximums for $k(x)$. The range is $\left[\dfrac{1}{3}, \infty\right)$.

19. Continuous



$[-7, 3]$ by $[-12, 8]$

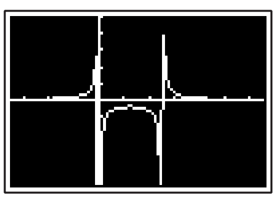
20. Continuous



$[-5, 5]$ by $[-8, 12]$

21. (a) $x^2 - 5x \neq 0, x(x - 5) \neq 0$, so $x \neq 0$ and $x \neq 5$. We expect vertical asymptotes at $x = 0$ and $x = 5$.

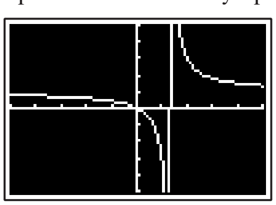
(b) $y = 0$



$[-7, 13]$ by $[-10, 10]$

22. (a) $x - 4 \neq 0, x \neq 4$, so we expect a vertical asymptote at $x = 4$.

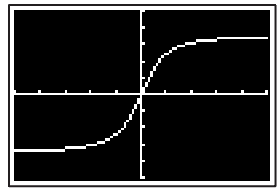
(b) Since $\lim\limits_{x\to\infty} \dfrac{3x}{x - 4} = 3$ and $\lim\limits_{x\to-\infty} \dfrac{3x}{x - 4} = 3$, we also expect a horizontal asymptote at $y = 3$.



$[-15, 15]$ by $[-15, 15]$

23. (a) None

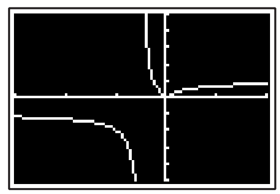
(b) Since $\lim\limits_{x\to\infty} \dfrac{7x}{\sqrt{x^2 + 10}} = 7$ and $\lim\limits_{x\to-\infty} \dfrac{7x}{\sqrt{x^2 + 10}} = -7$, we expect horizontal asymptotes at $y = 7$ and $y = -7$.



$[-15, 15]$ by $[-10, 10]$

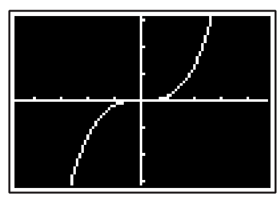
24. (a) $x + 1 \neq 0, x \neq -1$, so we expect a vertical asymptote at $x = -1$.

(b) $\lim\limits_{x\to\infty} \dfrac{|x|}{x + 1} = 1$ and $\lim\limits_{x\to-\infty} \dfrac{|x|}{x + 1} = -1$, so we can expect horizontal asymptotes at $y = 1$ and $y = -1$.



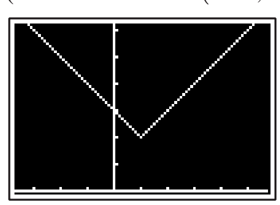
$[-6, 4]$ by $[-5, 5]$

25. $(-\infty, \infty)$



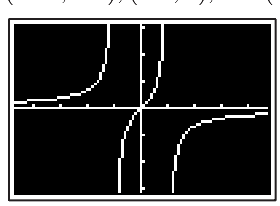
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

26. $|x - 1| = 0$ when $x = 1$, which is where the function's minimum occurs. y increases over the interval $[1, \infty)$. (Over the interval $(-\infty, 1]$, it is decreasing.)



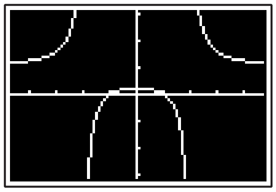
$[-3.7, 5.7]$ by $[0, 6.2]$

27. As the graph illustrates, y is increasing over the intervals $(-\infty, -1), (-1, 1)$, and $(1, \infty)$.



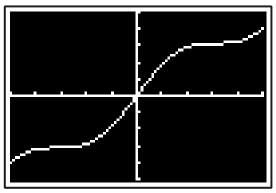
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

28. As the graph illustrates, y is increasing over the intervals $(-\infty, -2)$ and $(-2, 0]$.



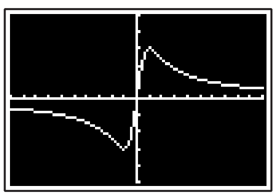
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

29. $-1 \le \sin x \le 1$, but $-\infty < x < \infty$, so $f(x)$ is not bounded.



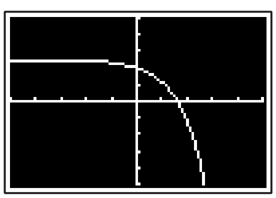
$[-5, 5]$ by $[-5, 5]$

30. $g(x) = 3$ at $x = 1$, a maximum and $g(x) = -3$, a minimum, at $x = -1$. It is bounded.



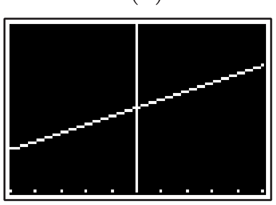
$[-10, 10]$ by $[-5, 5]$

31. $e^x > 0$ for all x, so $-e^x < 0$ and $5 - e^x < 5$ for all x. $h(x)$ is bounded above.



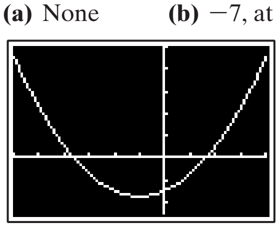
$[-5, 5]$ by $[-10, 10]$

32. The function is linear with slope $\frac{1}{1000}$ and y-intercept 1000. Thus $k(x)$ is not bounded.



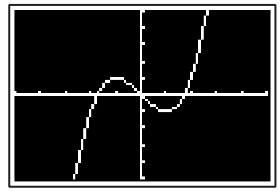
$[-5, 5]$ by $[-999.99, 1000.01]$

33. **(a)** None **(b)** -7, at $x = -1$



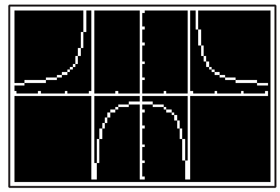
$[-6, 4]$ by $[-10, 20]$

34. **(a)** 2, at $x = -1$ **(b)** -2, at $x = 1$



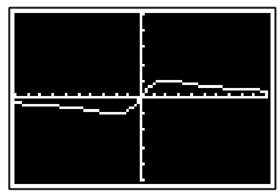
$[-5, 5]$ by $[-10, 10]$

35. **(a)** -1, at $x = 0$ **(b)** None



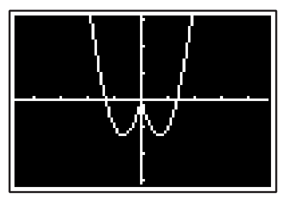
$[-5, 5]$ by $[-10, 10]$

36. **(a)** 1, at $x = 2$ **(b)** -1, at $x = -2$



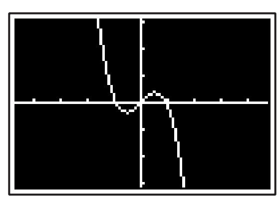
$[-10, 10]$ by $[-5, 5]$

37. The function is even since it is symmetrical about the y-axis.



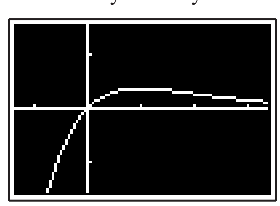
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

38. Since the function is symmetrical about the origin, it is odd.



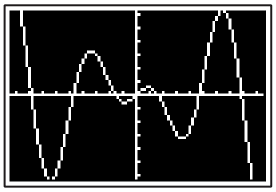
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

39. Since no symmetry is exhibited, the function is neither.



$[-1.35, 3.35]$ by $[-1.55, 1.55]$

40. Since the function is symmetrical about the origin, it is odd.



$[-9.4, 9.4]$ by $[-6.2, 6.2]$

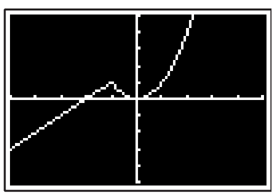
41. $x = 2y + 3, 2y = x - 3, y = \dfrac{x-3}{2}$, so
$f^{-1}(x) = \dfrac{x-3}{2}$.

42. $x = \sqrt[3]{y-8}, x^3 = y - 8, y = x^3 + 8$, so
$f^{-1}(x) = x^3 + 8$.

43. $x = \dfrac{2}{y}, xy = 2, y = \dfrac{2}{x}$, so $f^{-1}(x) = \dfrac{2}{x}$.

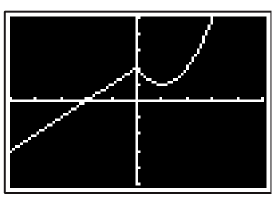
44. $x = \dfrac{6}{y+4}, (y+4)x = 6, xy + 4x = 6, xy = 6 - 4x$,
$y = \dfrac{6-4x}{x}$, so $f^{-1}(x) = \dfrac{6}{x} - 4$.

45.



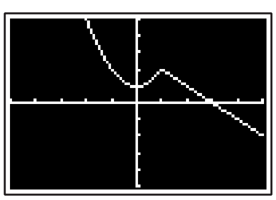
$[-5, 5]$ by $[-5, 5]$

46.



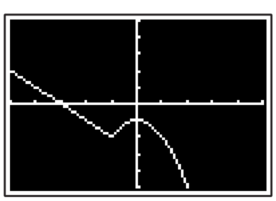
$[-5, 5]$ by $[-5, 5]$

47.



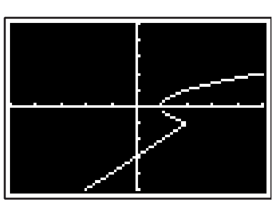
$[-5, 5]$ by $[-5, 5]$

48.



$[-5, 5]$ by $[-5, 5]$

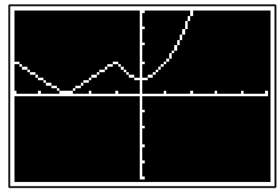
49.



$[-5, 5]$ by $[-5, 5]$

50. No

51.



$[-5, 5]$ by $[-5, 5]$

52. $f(x) = \begin{cases} x + 3 \text{ if } x \le -1 \\ x^2 + 1 \text{ if } x > -1 \end{cases}$

53. $(f \circ g)(x) = f(g(x)) = f(x^2 - 4) = \sqrt{x^2 - 4}$.
Since $x^2 - 4 \ge 0, x^2 \ge 4, x \le -2$ or $x \ge 2$.
The domain is $(-\infty, -2] \cup [2, \infty)$.

54. $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 4 =$
$x - 4$. Since $\sqrt{x} \ge 0, x \ge 0$. The domain is $[0, \infty)$.

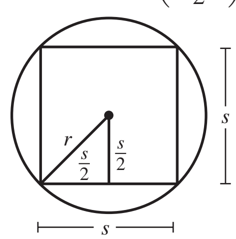
55. $(f \cdot g)(x) = f(x) \cdot g(x) = \sqrt{x} \cdot (x^2 - 4)$.
Since $\sqrt{x} \ge 0$, the domain is $[0, \infty)$.

56. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x}}{x^2 - 4}$ Since $x^2 - 4 \ne 0$,
$(x + 2)(x - 2) \ne 0, x \ne -2, x \ne 2$. Also since
$\sqrt{x} \ge 0, x \ge 0$. The domain is $[0, 2) \cup (2, \infty)$.

57. $\lim_{x \to \infty} \sqrt{x} = \infty$. (Large negative values are not in the domain.)

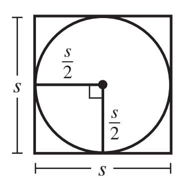
58. $\lim_{x \to \pm\infty} \sqrt{x^2 - 4} = \infty$. (The graph resembles the line $y = x$.)

59. $r^2 = \left(\dfrac{s}{2}\right)^2 + \left(\dfrac{s}{2}\right)^2 = \dfrac{2s^2}{4}, \quad r = \sqrt{\dfrac{2s^2}{4}} = \dfrac{s\sqrt{2}}{2}$.
The area of the circle is
$A = \pi r^2 = \pi\left(\dfrac{s\sqrt{2}}{2}\right)^2 = \dfrac{2\pi s^2}{4} = \dfrac{\pi s^2}{2}$

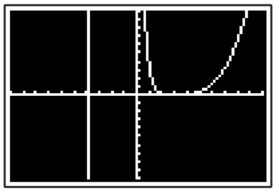


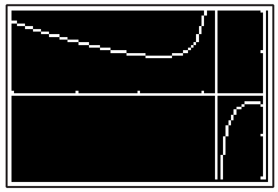
$r = \dfrac{s\sqrt{2}}{2}$

60. $A = \pi r^2 = \pi\left(\dfrac{s}{2}\right)^2 = \dfrac{\pi s^2}{4}$



Grapher:


[−10, 10] by [−10, 10]


[−20, 0] by [−1000, 1000]

73. (a) $|x - 3| < 1/3 \Rightarrow |3x - 9| < 1 \Rightarrow$
$|3x - 5 - 4| < 1 \Rightarrow |f(x) - 4| < 1$.
For example:
$|f(x) - 4| = |(3x - 5) - 4| = |3x - 9|$
$= 3|x - 3| < 3\left(\frac{1}{3}\right) = 1$

(b) If x stays within the dashed vertical lines, $f(x)$ will stay within the dashed horizontal lines. For the example in part (a), the graph shows that for $\frac{8}{3} < x < \frac{10}{3}$ $\left(\text{that is, } |x - 3| < \frac{1}{3}\right)$, we have $3 < f(x) < 5$ (that is, $|f(x) - 4| < 1$).

(c) $|x - 3| < 0.01 \Rightarrow |3x - 9| < 0.03 \Rightarrow$
$|3x - 5 - 4| < 0.03 \Rightarrow |f(x) - 4| < 0.03$. The dashed lines would be closer to $x = 3$ and $y = 4$.

74. When $x^2 - 4 \geq 0$, $y = 1$, and when $x^2 - 4 \ngeq 0$, $y = 0$.

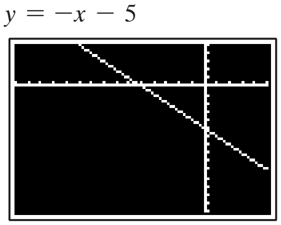
75. One possible answer: Given $0 < a < b$, multiplying both sides of $a < b$ by a gives $a^2 < ab$; multiplying by b gives $ab < b^2$. Then, by the transitive property of inequality, we have $a^2 < b^2$.

76. One possible answer: Given $0 < a < b$, multiplying both sides of $a < b$ by $\frac{1}{ab}$ gives $\frac{1}{b} < \frac{1}{a}$, which is equivalent to $\frac{1}{a} > \frac{1}{b}$.

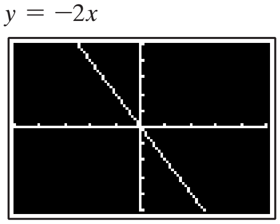
■ Chapter 2 Review

For #1 and 2, first find the slope of the line. Then use algebra to put into $y = mx + b$ format.

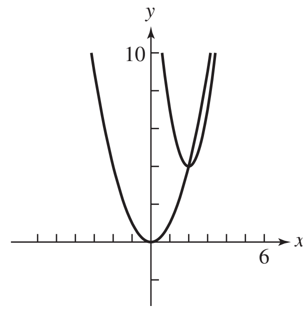
1. $m = \dfrac{-9 - (-2)}{4 - (-3)} = \dfrac{-7}{7} = -1$, $(y + 9) = -1(x - 4)$, $y = -x - 5$


[−15, 5] by [−15, 5]

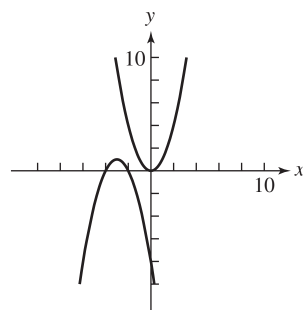
2. $m = \dfrac{-2 - 6}{1 - (-3)} = \dfrac{-8}{4} = -2$, $(y + 2) = -2(x - 1)$, $y = -2x$


[−5, 5] by [−5, 5]

3. Starting from $y = x^2$, translate right 2 units and vertically stretch by 3 (either order), then translate up 4 units.



4. Starting from $y = x^2$, translate left 3 units and reflect across x-axis (either order), then translate up 1 unit.



5. Vertex: $(-3, 5)$; axis: $x = -3$

6. Vertex: $(5, -7)$; axis: $x = 5$

7. $f(x) = -2(x^2 + 8x) - 31$
$= -2(x^2 + 8x + 16) + 32 - 31 = -2(x + 4)^2 + 1$;
Vertex: $(-4, 1)$; axis: $x = -4$

8. $g(x) = 3(x^2 - 2x) + 2 = 3(x^2 - 2x + 1) - 3 + 2 = 3(x - 1)^2 - 1$; Vertex: $(1, -1)$; axis: $x = 1$

For #9–12, use the form $y = a(x - h)^2 + k$, where (h, k), the vertex, is given.

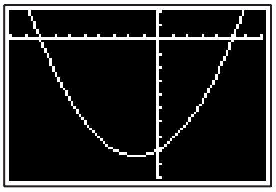
9. $h = -2$ and $k = -3$ are given, so $y = a(x + 2)^2 - 3$.
Using the point $(1, 2)$, we have $2 = 9a - 3$, so $a = \frac{5}{9}$:
$y = \frac{5}{9}(x + 2)^2 - 3$.

10. $h = -1$ and $k = 1$ are given, so $y = a(x + 1)^2 + 1$.
Using the point $(3, -2)$, we have $-2 = 16a + 1$, so
$a = -\frac{3}{16}$: $y = -\frac{3}{16}(x + 1)^2 + 1$.

11. $h = 3$ and $k = -2$ are given, so $y = a(x - 3)^2 - 2$.
Using the point $(5, 0)$, we have $0 = 4a - 2$, so $a = \frac{1}{2}$:
$y = \frac{1}{2}(x - 3)^2 - 2$.

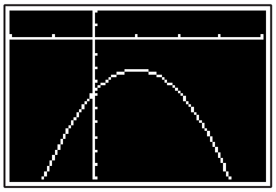
12. $h = -4$ and $k = 5$ are given, so $y = a(x + 4)^2 + 5$.
Using the point $(0, -3)$, we have $-3 = 16a + 5$,
so $a = -\frac{1}{2}$; $y = -\frac{1}{2}(x + 4)^{-2} + 5$.

13.



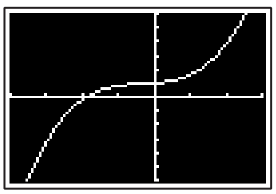
$[-10, 7]$ by $[-50, 10]$

14.



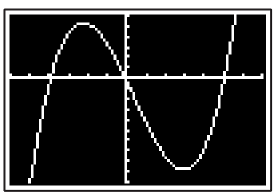
$[-2, 4]$ by $[-50, 10]$

15.



$[-4, 3]$ by $[-30, 30]$

16.



$[-6, 7]$ by $[-50, 30]$

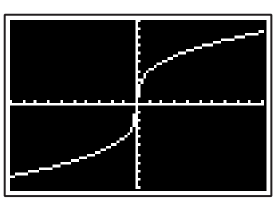
17. $S = kr^2 \quad (k = 4\pi)$

18. $F = \dfrac{k}{d^2} \quad (k = \text{gravitational constant})$

19. The force F needed varies directly with the distance x from its resting position, with constant of variation k.

20. The area of a circle A varies directly with the square of its radius.

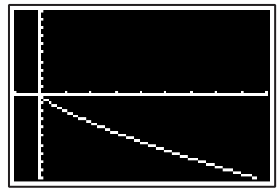
21. $k = 4$, $a = \dfrac{1}{3}$. In Quadrant I, $f(x)$ is increasing and concave down since $0 < a < 1$.



$[-10, 10]$ by $[-10, 10]$

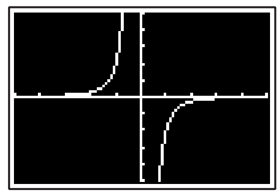
$f(-x) = 4(-x)^{1/3} = -4x^{1/3} = -f(x)$, so f is odd.

22. $k = -2$, $a = \dfrac{3}{4}$. In Quadrant IV, $f(x)$ is decreasing and concave up since $0 < a < 1$. f is not defined for $x < 0$.



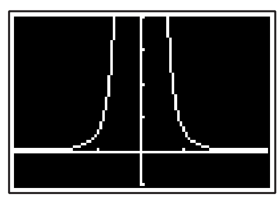
$[-1, 9]$ by $[-10, 10]$

23. $k = -2$, $a = -3$. In Quadrant IV, f is increasing and concave down. $f(-x) = -2(-x)^{-3} = \dfrac{-2}{(-x)^3} = \dfrac{-2}{-x^3}$ $= \dfrac{2}{x^3} = 2x^{-3} = -f(x)$, so f is odd.



$[-5, 5]$ by $[-5, 5]$

24. $k = \dfrac{2}{3}$, $a = -4$. In Quadrant I, $f(x)$ is decreasing and concave up. $f(-x) = \dfrac{2}{3}(-x)^{-4} = \dfrac{2}{3} \cdot \dfrac{1}{(-x)^4}$ $= \dfrac{2}{3x^4} = \dfrac{2}{3}x^{-4} = f(x)$, so f is even.



$[-3, 3]$ by $[-1, 4]$

25. $\dfrac{2x^3 - 7x^2 + 4x - 5}{x - 3} = 2x^2 - x + 1 - \dfrac{2}{x - 3}$

$$\begin{array}{r}
2x^2 - x + 1 \phantom{{}- 5} \\
x - 3 \,\big)\, \overline{2x^3 - 7x^2 + 4x - 5} \\
2x^3 - 6x^2 \phantom{{}+ 4x - 5} \\
\hline
-x^2 + 4x \phantom{{}- 5} \\
-x^2 + 3x \phantom{{}- 5} \\
\hline
x - 5 \\
x - 3 \\
\hline
-2
\end{array}$$

26. $\dfrac{x^4 + 3x^3 + x^2 - 3x + 3}{x + 2} = x^3 + x^2 - x - 1 + \dfrac{5}{x + 2}$

$$\begin{array}{r}
x^3 + x^2 - x - 1 \\
x + 2\,\overline{)\,x^4 + 3x^3 + x^2 - 3x + 3} \\
\underline{x^4 + 2x^3 \phantom{{}+ x^2 - 3x + 3}} \\
x^3 + x^2 \phantom{{}- 3x + 3} \\
\underline{x^3 + 2x^2 \phantom{{}- 3x + 3}} \\
-x^2 - 3x \phantom{{}+ 3} \\
\underline{-x^2 - 2x \phantom{{}+ 3}} \\
-x + 3 \\
\underline{-x - 2} \\
5
\end{array}$$

27. $\dfrac{2x^4 - 3x^3 + 9x^2 - 14x + 7}{x^2 + 4}$
$= 2x^2 - 3x + 1 + \dfrac{-2x + 3}{x^2 + 4}$

$$\begin{array}{r}
2x^2 - 3x + 1 \\
x^2 + 4\,\overline{)\,2x^4 - 3x^3 + 9x^2 - 14x + 7} \\
\underline{2x^4 \phantom{{}- 3x^3} + 8x^2 \phantom{{}- 14x + 7}} \\
-3x^3 + x^2 - 14x + 7 \\
\underline{-3x^3 \phantom{{}+ x^2} - 12x \phantom{{}+ 7}} \\
x^2 - 2x + 7 \\
\underline{x^2 \phantom{{}- 2x} + 4} \\
-2x + 3
\end{array}$$

28. $\dfrac{3x^4 - 5x^3 - 2x^2 + 3x - 6}{3x + 1} = x^3 - 2x^2 + 1 + \dfrac{-7}{3x + 1}$

$$\begin{array}{r}
x^3 - 2x^2 \phantom{{}+ 3x} + 1 \\
3x + 1\,\overline{)\,3x^4 - 5x^3 - 2x^2 + 3x - 6} \\
\underline{3x^4 + x^3 \phantom{{}- 2x^2 + 3x - 6}} \\
-6x^3 - 2x^2 + 3x - 6 \\
\underline{-6x^3 - 2x^2 \phantom{{}+ 3x - 6}} \\
3x - 6 \\
\underline{3x + 1} \\
-7
\end{array}$$

29. Remainder: $f(-2) = -39$

30. Remainder: $f(3) = -2$

31. Yes: 2 is a zero of the second polynomial.

32. No: $x = -3$ yields 1 from the second polynomial.

33.
$$\begin{array}{r|rrrr}
5 & 1 & -5 & 3 & 4 \\
 & & 5 & 0 & 15 \\ \hline
 & 1 & 0 & 3 & 19
\end{array}$$
Yes, $x = 5$ is an upper bound for the zeros of $f(x)$ because all entries on the bottom row are nonnegative.

34.
$$\begin{array}{r|rrrrr}
4 & 4 & -16 & 8 & 16 & -12 \\
 & & 16 & 0 & 32 & 192 \\ \hline
 & 4 & 0 & 8 & 48 & 180
\end{array}$$
Yes, $x = 4$ is an upper bound for the zeros of $f(x)$ because all entries on the bottom row are nonnegative.

35.
$$\begin{array}{r|rrrrr}
-3 & 4 & 4 & -15 & -17 & -2 \\
 & & -12 & 24 & -27 & 132 \\ \hline
 & 4 & -8 & 9 & -44 & 130
\end{array}$$
Yes, $x = -3$ is a lower bound for the zeros of $f(x)$ because all entries on the bottom row alternate signs.

36.
$$\begin{array}{r|rrrr}
-3 & 2 & 6 & 1 & -6 \\
 & & -6 & 0 & -3 \\ \hline
 & 2 & 0 & 1 & -9
\end{array}$$
Yes, $x = -3$ is a lower bound for the zeros of $f(x)$ because all entries on the bottom row alternate signs (remember that $0 = -0$).

37. Possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$,
or $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$; $-\frac{3}{2}$ and 2 are zeros.

38. Possible rational zeros: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$,
or $\pm 1, \pm 7, \pm\frac{1}{2}, \pm\frac{7}{2}, \pm\frac{1}{3}, \pm\frac{7}{3}, \pm\frac{1}{6}, \pm\frac{7}{6}$; $\frac{7}{3}$ is a zero.

39. $(1 + i)^3 = (1 + 2i + i^2)(1 + i) = (2i)(1 + i)$
$= -2 + 2i$

40. $(1 + 2i)^2(1 - 2i)^2 = [(1 + 2i)(1 - 2i)]^2 = (1 + 2^2)^2$
$= 25$

41. $i^{29} = i$

42. $\sqrt{-16} = 4i$

For #43–44, use the quadratic formula.

43. $x = \dfrac{6 \pm \sqrt{36 - 52}}{2} = \dfrac{6 \pm 4i}{2} = 3 \pm 2i$

44. $x = \dfrac{2 \pm \sqrt{4 - 16}}{2} = \dfrac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i$

45. (c) $f(x) = (x - 2)^2$ is a quadratic polynomial that has vertex $(2, 0)$ and y-intercept $(0, 4)$, so its graph must be graph (c).

46. (d) $f(x) = (x - 2)^3$ is a cubic polynomial that passes through $(2, 0)$ and $(0, -8)$, so its graph must be graph (d).

47. (b) $f(x) = (x - 2)^4$ is a quartic polynomial that passes through $(2, 0)$ and $(0, 16)$, so its graph must be graph (b).

48. (a) $f(x) = (x - 2)^5$ is a quintic polynomial that passes through $(2, 0)$ and $(0, -32)$, so its graph must be graph (a).

In #49–52, use a graph and the Rational Zeros Test to determine zeros.

49. Rational: 0 (multiplicity 2) — easily seen by inspection. Irrational: $5 \pm \sqrt{2}$ (using the quadratic formula, after taking out a factor of x^2). No non-real zeros.

50. Rational: ± 2. Irrational: $\pm\sqrt{3}$. No nonreal zeros. These zeros may be estimated from a graph, or by dividing $k(t)$ by $t - 2$ and $t + 2$ then applying the quadratic formula, or by using the quadratic formula on $k(t)$ to determine that $t^2 = \dfrac{7 \pm \sqrt{49 - 48}}{2}$, i.e., t^2 is 3 or 4.

51. Rational: none. Irrational: approximately -2.34, 0.57, 3.77. No non-real zeros.

52. Rational: none. Irrational: approximately −3.97, −0.19. Two non-real zeros.

53. The only rational zero is $-\frac{3}{2}$. Dividing by $x + \frac{3}{2}$ (below) leaves $2x^2 - 12x + 20$, which has zeros $\frac{12 \pm \sqrt{144 - 160}}{4} = 3 \pm i$. Therefore

$$f(x) = (2x + 3)[x - (3 - i)][x - (3 + i)] = (2x + 3)(x - 3 + i)(x - 3 - i).$$

−3/2	2	−9	2	30
		−3	18	−30
	2	−12	20	0

54. The only rational zero is $\frac{4}{5}$. Dividing by $x - \frac{4}{5}$ (below) leaves $5x^2 - 20x - 15$, which has zeros $\frac{20 \pm \sqrt{400 + 300}}{10} = 2 \pm \sqrt{7}$. Therefore

$$f(x) = (5x - 4)\left[x - \left(2 + \sqrt{7}\right)\right]\left[x - \left(2 - \sqrt{7}\right)\right] = (5x - 4)\left(x - 2 - \sqrt{7}\right)\left(x - 2 + \sqrt{7}\right).$$

4/5	5	−24	1	12
		4	−16	−12
	5	−20	−15	0

55. All zeros are rational: 1, −1, $\frac{2}{3}$, and $-\frac{5}{2}$. Therefore $f(x) = (3x - 2)(2x + 5)(x - 1)(x + 1)$; this can be confirmed by multiplying out the terms or graphing the original function and the factored form of the function.

56. Since all coefficients are real, $1 - 2i$ is also a zero. Dividing synthetically twice leaves the quadratic $x^2 - 6x + 10$, which has zeros $3 \pm i$.
$f(x) = [x - (1 + 2i)][x - (1 - 2i)][x - (3 + i)][x - (3 - i)] = (x - 1 - 2i)(x - 1 + 2i)(x - 3 - i)(x - 3 + i)$

1 + 2i	1	−8	27	−50	50
		1 + 2i	−11 − 12i	40 + 20i	−50
	1	−7 + 2i	16 − 12i	−10 + 20i	0
1 − 2i	1	−7 + 2i	16 − 12i	−10 + 20i	
		1 − 2i	−6 + 12i	10 − 20i	
	1	−6	10	0	

In #57–60, determine rational zeros (graphically or otherwise) and divide synthetically until a quadratic remains. If more real zeros remain, use the quadratic formula.

57. The only real zero is 2; dividing by $x - 2$ leaves the quadratic factor $x^2 + x + 1$, so $f(x) = (x - 2)(x^2 + x + 1)$.

2	1	−1	−1	−2
		2	2	2
	1	1	1	0

58. The only rational zero is −1; dividing by $x + 1$ leaves the quadratic factor $9x^2 - 12x - 1$, which has zeros $\frac{12 \pm \sqrt{144 + 36}}{18} = \frac{2}{3} \pm \frac{1}{3}\sqrt{5}$. Then $f(x) = (x + 1)(9x^2 - 12x - 1)$.

−1	9	−3	−13	−1
		−9	12	1
	9	−12	−1	0

59. The two real zeros are 1 and $\frac{3}{2}$; dividing by $x - 1$ and $x - \frac{3}{2}$ leaves the quadratic factor $2x^2 - 4x + 10$, so $f(x) = (2x - 3)(x - 1)(x^2 - 2x + 5)$.

1	2	−9	23	−31	15
		2	−7	16	−15
	2	−7	16	−15	0

3/2	2	−7	16	−15
		3	−6	15
	2	−4	10	0

60. The two real zeros are −1 and $-\frac{2}{3}$; dividing by $x + 1$ and $x + \frac{2}{3}$ leaves the quadratic factor $3x^2 - 12x + 15$, so $f(x) = (3x + 2)(x + 1)(x^2 - 4x + 5)$.

−1	3	−7	−3	17	10
		−3	10	−7	−10
	3	−10	7	10	0

−2/3	3	−10	7	10
		−2	8	−10
	3	−12	15	0

61. $(x - \sqrt{5})(x + \sqrt{5})(x - 3) = x^3 - 3x^2 - 5x + 15$. Other answers may be found by multiplying this polynomial by any real number.

62. $(x + 3)^2 = x^2 + 6x + 9$ (This may be multiplied by any real number.)

63. $(x - 3)(x + 2)(3x - 1)(2x + 1) = 6x^4 - 5x^3 - 38x^2 - 5x + 6$ (This may be multiplied by any real number.)

64. The third zero must be $1 - i$: $(x - 2)(x - 1 - i)(x - 1 + i) = x^3 - 4x^2 + 6x - 4$ (This may be multiplied by any real number.)

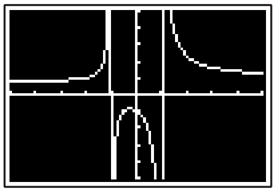
65. $(x + 2)^2(x - 4)^2 = x^4 - 4x^3 - 12x^2 + 32x + 64$ (This may be multiplied by any real number.)

66. The third zero must be $2 + i$, so $f(x) = a(x + 1)(x - 2 - i)(x - 2 + i)$. Since $f(2) = 6$, $a = 2$:
$f(x) = 2(x + 1)(x - 2 - i)(x - 2 + i) = 2x^3 - 6x^2 + 2x + 10$.

67. $f(x) = -1 + \frac{2}{x - 5}$; translate right 5 units and vertically stretch by 2 (either order), then translate down 1 unit. Horizontal asymptote: $y = -1$; vertical asymptote: $x = 5$.

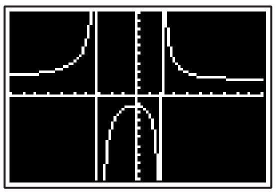
68. $f(x) = 3 - \frac{1}{x - 2}$; translate left 2 units and reflect across x-axis (either order), then translate up 3 units. Horizontal asymptote: $y = 3$; vertical asymptote: $x = -2$.

69. Asymptotes: $y = 1$, $x = -1$, and $x = 1$.
Intercept: $(0, -1)$.



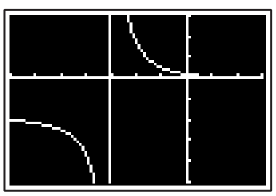
$[-5, 5]$ by $[-5, 5]$

70. Asymptotes: $y = 2$, $x = -3$, and $x = 2$.
Intercept: $\left(0, -\frac{7}{6}\right)$.



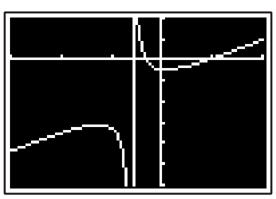
$[-10, 10]$ by $[-10, 10]$

71. End-behavior asymptote: $y = x - 7$.
Vertical asymptote: $x = -3$. Intercept: $\left(0, \frac{5}{3}\right)$.



$[-7, 3]$ by $[-50, 30]$

72. End-behavior asymptote: $y = x - 6$.
Vertical asymptote: $x = -3$. Intercepts: approx. $(-1.54, 0)$, $(4.54, 0)$, and $\left(0, -\frac{7}{3}\right)$.



$[-15, 10]$ by $[-30, 10]$

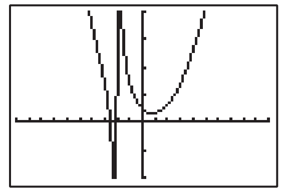
73. $f(x) = \dfrac{x^3 + x^2 - 2x + 5}{x + 2}$ has only one x-intercept, and we can use the graph to show that it is about -2.552. The y-intercept is $f(0) = 5/2$. The denominator is zero when $x = -2$, so the vertical asymptote is $x = -2$. Because we can rewrite $f(x)$ as

$$f(x) = \frac{x^3 + x^2 - 2x + 5}{x + 2} = x^2 - x + \frac{5}{x + 2},$$

we know that the end-behavior asymptote is $y = x^2 - x$. The graph supports this information and allows us to conclude that

$\lim_{x \to -2^-} = -\infty$, $\lim_{x \to -2^+} = \infty$.

The graph also shows a local minimum of about 1.63 at about $x = 0.82$.



$[-10, 10]$ by $[-10, 20]$

y-intercept: $\left(0, \frac{5}{2}\right)$

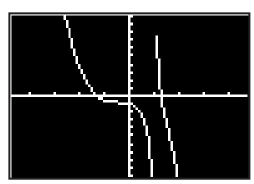
x-intercept: $(-2.55, 0)$

Domain: All $x \neq -2$
Range: $(-\infty, \infty)$
Continuity: All $x \neq -2$
Increasing on $[0.82, \infty)$
Decreasing on $(-\infty, -2)$, $(-2, 0.82]$
Not symmetric.
Unbounded.
Local minimum: $(0.82, 1.63)$
No horizontal asymptote. End-behavior asymptote: $y = x^2 - x$
Vertical asymptote: $x = -2$.
End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = \infty$

74. $f(x) = \dfrac{-x^4 + x^2 + 1}{x - 1}$ has two x-intercepts, and we can use the graph to show that they are about -1.27 and 1.27. The y-intercept is $f(0) = -1$. The denominator is zero when $x = 1$, so the vertical asymptote is $x = 1$. Because we can rewrite $f(x)$ as

$$f(x) = \frac{-x^4 + x^2 + 1}{x - 1} = -x^3 - x^2 + \frac{1}{x - 1},$$

we know that the end-behavior asymptote is $y = -x^3 - x^2$. The graph supports this information and allows us to conclude that $\lim_{x \to 1^-} = -\infty$ and $\lim_{x \to 1^+} = \infty$. The graph shows no local extrema.



$[-4.7, 4.7]$ by $[-10, 10]$

y-intercept: $(0, 1)$

x-intercepts: $(-1.27, 0)$, $(1.27, 0)$

Domain: All $x \neq 1$
Range: $(-\infty, \infty)$
Continuity: All $x \neq 1$
Never increasing
Decreasing on $(-\infty, 1)$, $(1, \infty)$
No symmetry.
Unbounded.
No local extrema.
No horizontal asymptote. End-behavior asymptote: $y = -x^3 - x^2$
Vertical asymptote: $x = 1$
End behavior: $\lim_{x \to -\infty} f(x) = \infty$; $\lim_{x \to \infty} f(x) = -\infty$

75. Multiply by x: $2x^2 - 11x + 12 = 0$, so $x = \frac{3}{2}$ or $x = 4$.

76. Multiply by $(x + 2)(x - 3) = x^2 - x - 6$: $x(x - 3) + 5(x + 2) = 25$, or $x^2 + 2x - 15 = 0$, so $x = -5$ or $x = 3$. The latter is extraneous; the only solution is $x = -5$.

For #77–78, find the zeros of $f(x)$ and then determine where the function is positive or negative by creating a sign chart.

77. $f(x) = (x - 3)(2x + 5)(x + 2)$, so the zeros of $f(x)$ are $x = \left\{-\frac{5}{2}, -2, 3\right\}$.

(–)(–)(–) Negative | $-\frac{5}{2}$ | (–)(+)(–) Positive | -2 | (–)(+)(+) Negative | 3 | (+)(+)(+) Positive x

As our sign chart indicates, $f(x) < 0$ on the interval $\left(-\infty, -\frac{5}{2}\right) \cup (-2, 3)$.

78. $f(x) = (x - 2)^2(x + 4)(3x + 1)$, so the zeros of $f(x)$ are $x = \left\{-4, -\frac{1}{3}, 2\right\}$.

(+)(–)(–) Positive | -4 | (+)(+)(–) Negative | $-\frac{1}{3}$ | (+)(+)(+) Positive | 2 | (+)(+)(+) Positive x

As our sign chart indicates, $f(x) \geq 0$ on the interval $(-\infty, -4] \cup \left[-\frac{1}{3}, \infty\right)$.

79. Zeros of numerator and denominator: -3, -2, and 2. Choose -4, -2.5, 0, and 3; $\frac{x + 3}{x^2 - 4}$ is positive at -2.5 and 3, and equals 0 at -3, so the solution is $[-3, -2) \cup (2, \infty)$.

80. $\frac{x^2 - 7}{x^2 - x - 6} - 1 = \frac{x - 1}{x^2 - x - 6}$. Zeros of numerator and denominator: -2, 1, and 3. Choose -3, 0, 2, and 4; $\frac{x - 1}{x^2 - x - 6}$ is negative at -3 and 2, so the solution is $(-\infty, -2) \cup (1, 3)$.

81. Since the function is always positive, we need only worry about the equality $(2x - 1)^2|x + 3| = 0$. By inspection, we see this holds true only when $x = \left\{-3, \frac{1}{2}\right\}$.

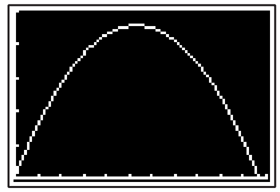
82. $\sqrt{x + 3}$ exists only when $x \geq -3$, so we are concerned only with the interval $(-3, \infty)$. Further $|x - 4|$ is always 0 or positive, so the only possible value for a sign change is $x = 1$. For $-3 < x < 1$, $\frac{(x - 1)|x - 4|}{\sqrt{x + 3}}$ is negative, and for $1 < x < 4$ or $4 < x < \infty$, $\frac{(x - 1)|x + 4|}{\sqrt{x + 3}}$ is positive. So the solution is $(1, 4) \cup (4, \infty)$.

83. Synthetic division reveals that we *cannot* conclude that 5 is an upper bound (since there are both positive and negative numbers on the bottom row), while -5 *is* a lower bound (because all numbers on the bottom row alternate signs). Yes, there is another zero (at $x \approx 10.0002$).

5	1	−10	−3	28	20	−2
		5	−25	−140	−560	−2700
	1	−5	−28	−112	−540	−2702

−5	1	−10	−3	28	20	−2
		−5	75	−360	1660	−8400
	1	−15	72	−332	1680	−8402

84. (a) $h = -16t^2 + 170t + 6$



[0, 11] by [0, 500]

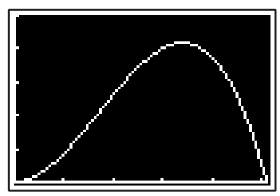
(b) When $t \approx 5.3125$, $h \approx 457.5625$.

(c) The rock will hit the ground after about 10.66 sec.

85. (a) $V = $ (height)(width)(length) $= x(30 - 2x)(70 - 2x)$ in.^{3}

(b) Either $x \approx 4.57$ or $x \approx 8.63$ in.

86. (a) & **(b)**



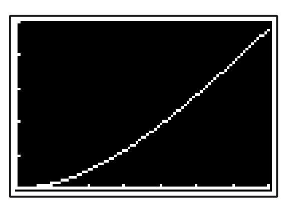
[0, 255] by [0, 2.5]

(c) When $d \approx 170$ ft, $s \approx 2.088$ ft.

(d) One possibility: The beam may taper off (become thinner) from west to east — e.g., perhaps it measures 8 in. by 8 in. at the west end, but only 7 in. by 7 in. on the east end. Then we would expect the beam to bend more easily closer to the east end (though not at the extreme east end, since it is anchored to the piling there). Another possibility: The two pilings are made of different materials.

87. (a) The tank is made up of a cylinder, with volume $\pi x^2(140 - 2x)$, and a sphere, with volume $\frac{4}{3}\pi x^3$. Thus, $V = \frac{4}{3}\pi x^3 + \pi x^2(140 - 2x)$.

(b)



[0, 70] by [0, 1,500,000]

of the others. Rates of growth in populations (esp. human populations) tend to fluctuate more than exponential models suggest. Of course, an exponential model also fits well in compound interest situations where the interest rate is held constant, but there are many cases where interest rates change over time.

60. **(a)** Steve's balance will always remain \$1000, since interest is not added to it. Every year he receives 6% of that \$1000 in interest: 6% in the first year, then another 6% in the second year (for a total of $2 \cdot 6\% = 12\%$), then another 6% (totaling $3 \cdot 6\% = 18\%$), etc. After t years, he has earned $6t$% of the \$1000 investment, meaning that altogether he has $1000 + 1000 \cdot 0.06t = 1000(1 + 0.06t)$.

(b) The table is shown below; the second column gives values of $1000(1.06)^t$. The effects of annual compounding show up beginning in year 2.

Years	Not Compounded	Compounded
0	1000.00	1000.00
1	1060.00	1060.00
2	1120.00	1123.60
3	1180.00	1191.02
4	1240.00	1262.48
5	1300.00	1338.23
6	1360.00	1418.52
7	1420.00	1503.63
8	1480.00	1593.85
9	1540.00	1689.48
10	1600.00	1790.85

61. False. The limit, with continuous compounding, is $A = Pe^{rt} = 100\,e^{0.05} \approx \$105.13$.

62. True. The calculation of interest paid involves compounding, and the compounding effect is greater for longer repayment periods.

63. $A = P(1 + r/k)^{kt} = 2250(1 + 0.07/4)^{4(6)} \approx \$3412.00$. The answer is B.

64. Let $x = \text{APY}$. Then $1 + x = (1 + 0.06/12)^{12} \approx 1.0617$. So $x \approx 0.0617$. The answer is C.

65. $FV = R((1 + i)^n - 1)/i = 300((1 + 0.00375)^{240} - 1)/0.00375 \approx \$116{,}437.31$. The answer is E.

66. $R = PV\,i/(1 - (1 + i)^{-n})$
$= 120{,}000(0.0725/12)/(1 - (1 + 0.0725/12)^{-180})$
$\approx \$1095.44$.
The answer is A.

67. The last payment will be \$364.38.

68. One possible answer:

The answer is (c). This graph shows the loan balance decreasing at a fairly steady rate over time. By contrast, the early payments on a 30-year mortgage go mostly toward interest, while the late payments go mostly toward paying down the debt. So the graph of loan balance versus time for a 30-year mortgage at double the interest rate would start off nearly horizontal and more steeply decrease over time.

69. **(a)** Matching up with the formula $S = R\,\dfrac{(1 + i)^n - 1}{i}$, where $i = r/k$, with r being the rate and k being the number of payments per year, we find $r = 8\%$.

(b) $k = 12$ payments per year.

(c) Each payment is $R = \$100$.

70. **(a)** Matching up with the formula $A = R\,\dfrac{1 - (1 + i)^{-n}}{i}$, where $i = r/k$, with r being the rate and k being number of payments per year, we find $r = 8\%$.

(b) $k = 12$ payments per year.

(c) Each payment is $R = \$200$.

■ Chapter 3 Review

1. $f\left(\dfrac{1}{3}\right) = -3 \cdot 4^{1/3} = -3\sqrt[3]{4}$

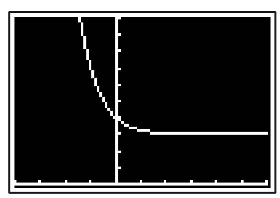
2. $f\left(-\dfrac{3}{2}\right) = 6 \cdot 3^{-3/2} = \dfrac{6}{\sqrt{27}} = \dfrac{2}{\sqrt{3}}$

For #3–4, recall that exponential functions have the form $f(x) = a \cdot b^x$.

3. $a = 3$, so $f(2) = 3 \cdot b^2 = 6$, $b^2 = 2$, $b = \sqrt{2}$, $f(x) = 3 \cdot 2^{x/2}$

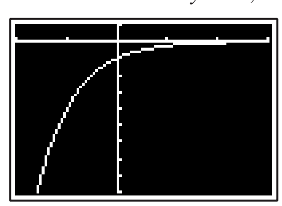
4. $a = 2$, so $f(3) = 2 \cdot b^3 = 1$, $b^3 = \dfrac{1}{2}$, $b = 2^{-1/3}$, $f(x) = 2 \cdot 2^{-x/3}$

5. $f(x) = 2^{-2x} + 3$ — starting from 2^x, horizontally shrink by $\dfrac{1}{2}$, reflect across y-axis, and translate up 3 units.



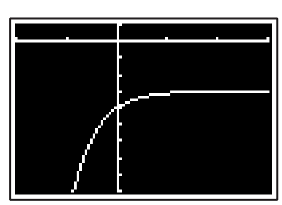
$[-4, 6]$ by $[0, 10]$

6. $f(x) = 2^{-2x}$ — starting from 2^x, horizontally shrink by $\dfrac{1}{2}$, reflect across the y-axis, reflect across x-axis.



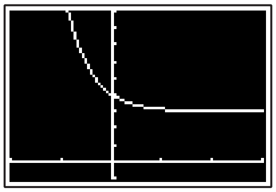
$[-2, 3]$ by $[9, -1]$

7. $f(x) = -2^{-3x} - 3$ — starting from 2^x, horizontally shrink by $\dfrac{1}{3}$, reflect across the y-axis, reflect across x-axis, translate down 3 units.



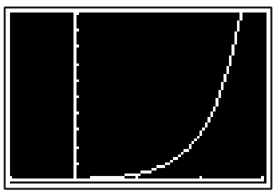
$[-2, 3]$ by $[9, -1]$

8. $f(x) = 2^{-3x} + 3$ — starting from 2^x, horizontally shrink by $\frac{1}{3}$, reflect across the y-axis, translate up 3 units.



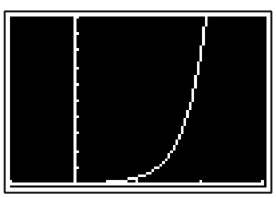
$[-2, 3]$ by $[-1, 9]$

9. Starting from e^x, horizontally shrink by $\frac{1}{2}$, then translate right $\frac{3}{2}$ units — or translate right 3 units, then horizontally shrink by $\frac{1}{2}$.



$[-1, 3]$ by $[0, 10]$

10. Starting from e^x, horizontally shrink by $\frac{1}{3}$, then translate right $\frac{4}{3}$ units — or translate right 4 units, then horizontally shrink by $\frac{1}{3}$.

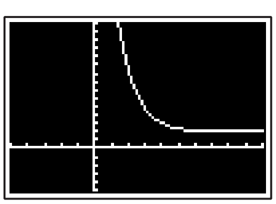


$[-1, 3]$ by $[0, 25]$

11. $f(0) = \dfrac{100}{5 + 3} = 12.5$, $\lim\limits_{x\to-\infty} f(x) = 0$, $\lim\limits_{x\to\infty} f(x) = 20$
y-intercept: $(0, 12.5)$; Asymptotes: $y = 0$ and $y = 20$

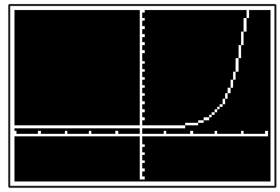
12. $f(0) = \dfrac{50}{5 + 2} = \dfrac{50}{7}$, $\lim\limits_{x\to-\infty} f(x) = 0$, $\lim\limits_{x\to\infty} f(x) = 10$
y-intercept: $\left(0, \dfrac{50}{7}\right) \approx (0, 7.14)$
Asymptotes: $y = 0$, $y = 10$

13. It is an exponential decay function.
$\lim\limits_{x\to\infty} f(x) = 2$, $\lim\limits_{x\to-\infty} f(x) = \infty$



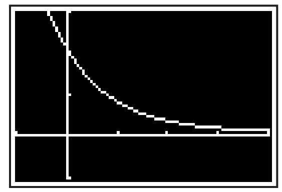
$[-5, 10]$ by $[-5, 15]$

14. Exponential growth function
$\lim\limits_{x\to\infty} f(x) = \infty$, $\lim\limits_{x\to-\infty} f(x) = 1$



$[-5, 5]$ by $[-5, 15]$

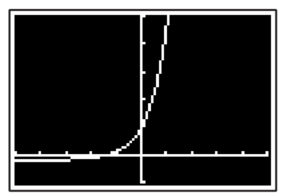
15.



$[-1, 4]$ by $[-10, 30]$

Domain: $(-\infty, \infty)$
Range: $(1, \infty)$
Continuous
Always decreasing
Not symmetric
Bounded below by $y = 1$, which is also the only asymptote
No local extrema
$\lim\limits_{x\to\infty} f(x) = 1$, $\lim\limits_{x\to-\infty} f(x) = \infty$

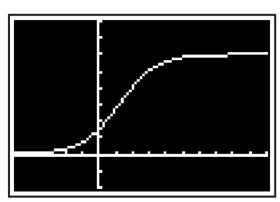
16.



$[-5, 5]$ by $[-10, 50]$

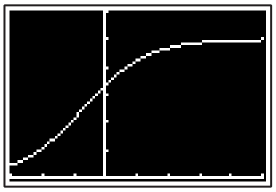
Domain: $(-\infty, \infty)$
Range: $(-2, \infty)$
Continuous
Always increasing
Not symmetric
Bounded below by $y = -2$, which is also the only asymptote
No local extrema
$\lim\limits_{x\to\infty} g(x) = \infty$, $\lim\limits_{x\to-\infty} g(x) = -2$

17.



$[-5, 10]$ by $[-2, 8]$

Domain: $(-\infty, \infty)$
Range: $(0, 6)$
Continuous
Increasing
Symmetric about $(1.20, 3)$
Bounded above by $y = 6$ and below by $y = 0$, the two asymptotes
No extrema
$\lim\limits_{x\to\infty} f(x) = 6$, $\lim\limits_{x\to-\infty} f(x) = 0$

18. 

$[-300, 500]$ by $[0, 30]$

Domain: $(-\infty, \infty)$
Range: $(0, 25)$
Continuous
Always increasing
Symmetric about $(-69.31, 12.5)$
Bounded above by $y = 25$ and below by $y = 0$, the two asymptotes
No local extrema
$\lim_{x\to\infty} g(x) = 25$, $\lim_{x\to-\infty} g(x) = 0$

For #19–22, recall that exponential functions are of the form $f(x) = a \cdot (1 + r)^{kx}$.

19. $a = 24$, $r = 0.053$, $k = 1$; so $f(x) = 24 \cdot 1.053^x$, where $x =$ days.

20. $a = 67{,}000$, $r = 0.0167$, $k = 1$, so $f(x) = 67{,}000 \cdot 1.0167^x$, where $x =$ years.

21. $a = 18$, $r = 1$, $k = \frac{1}{21}$, so $f(x) = 18 \cdot 2^{x/21}$, where $x =$ days.

22. $a = 117$, $r = -\frac{1}{2}$, $k = \frac{1}{262}$, so

$f(x) = 117 \cdot \left(\frac{1}{2}\right)^{x/262} = 117 \cdot 2^{-x/262}$, where $x =$ hours.

For #23–26, recall that logistic functions are expressed in $f(x) = \dfrac{c}{1 + ae^{-bx}}$.

23. $c = 30$, $a = 1.5$, so $f(2) = \dfrac{30}{1 + 1.5e^{-2b}} = 20$,

$30 = 20 + 30e^{-2b}$, $30e^{-2b} = 10$, $e^{-2b} = \frac{1}{3}$,

$-2b \ln e = \ln \frac{1}{3} \approx -1.0986$, so $b \approx 0.55$.

Thus, $f(x) = \dfrac{30}{1 + 1.5e^{-0.55x}}$

24. $c = 20$, $a \approx 2.33$, so $f(3) = \dfrac{20}{1 + 2.33e^{-3b}} = 15$,

$20 = 15 + 35e^{-3b}$, $35e^{-3b} = 5$, $e^{-3b} = \frac{1}{7}$,

$-3b \ln e = \ln \frac{1}{7} \approx -1.9459$, so $b \approx 0.65$.

Thus, $f(x) = \dfrac{20}{1 + 2.33e^{-0.65x}}$

25. $c = 20$, $a = 3$, so $f(3) = \dfrac{20}{1 + 3e^{-3b}} = 10$,

$20 = 10 + 30e^{-3b}$, $30e^{-3b} = 10$, $e^{-3b} = \frac{10}{30} = \frac{1}{3}$,

$-3b \ln e = \ln \frac{1}{3} \approx -1.0986$, so $b \approx 0.37$.

Thus, $f(x) \approx \dfrac{20}{1 + 3e^{-0.37x}}$

26. $c = 44$, $a = 3$, so $f(5) = \dfrac{44}{1 + 3e^{-5b}} = 22$,

$44 = 22 + 66e^{-5b}$, $66e^{-5b} = 22$, $e^{-5b} = \frac{22}{66} = \frac{1}{3}$,

$-5b \ln e = \ln \frac{1}{3} \approx -1.0986$, so $b \approx 0.22$.

Thus, $f(x) \approx \dfrac{44}{1 + 3e^{-0.22x}}$

27. $\log_2 32 = \log_2 2^5 = 5 \log_2 2 = 5$

28. $\log_3 81 = \log_3 3^4 = 4 \log_3 3 = 4$

29. $\log \sqrt[3]{10} = \log 10^{\frac{1}{3}} = \frac{1}{3} \log 10 = \frac{1}{3}$

30. $\ln \dfrac{1}{\sqrt{e^7}} = \ln e^{-\frac{7}{2}} = -\frac{7}{2} \ln e = -\frac{7}{2}$

31. $x = 3^5 = 243$

32. $x = 2^y$

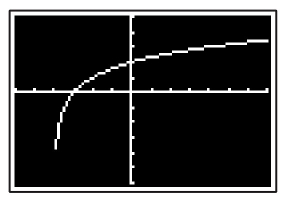
33. $\left(\dfrac{x}{y}\right) = e^{-2}$

$x = \dfrac{y}{e^2}$

34. $\left(\dfrac{a}{b}\right) = 10^{-3}$

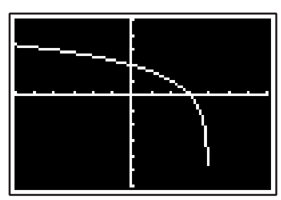
$a = \dfrac{b}{1000}$

35. Translate left 4 units.



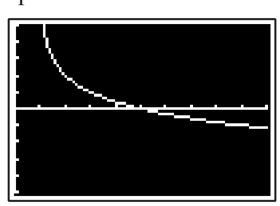
$[-6, 7]$ by $[-6, 5]$

36. Reflect across y-axis and translate right 4 units — or translate left 4 units, then reflect across y-axis.



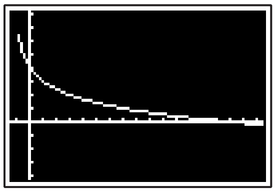
$[-6, 7]$ by $[-6, 5]$

37. Translate right 1 unit, reflect across x-axis, and translate up 2 units.



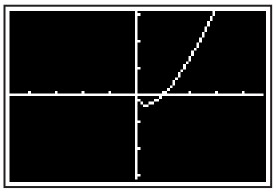
$[0, 10]$ by $[-5, 5]$

38. Translate left 1 unit, reflect across x-axis, and translate up 4 units.



$[-1.4, 17.4]$ by $[-4.2, 8.2]$

39.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Domain: $(0, \infty)$

Range: $\left[-\frac{1}{e}, \infty\right) \approx [-0.37, \infty)$

Continuous

Decreasing on $(0, 0.37]$; increasing on $[0.37, \infty)$

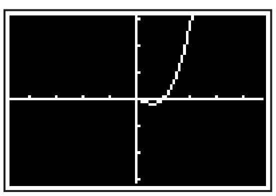
Not symmetric

Bounded below

Local minimum at $\left(\frac{1}{e}, -\frac{1}{e}\right)$

$\lim_{x\to\infty} f(x) = \infty$

40.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Domain: $(0, \infty)$

Range: $[-0.18, \infty)$

Continuous

Decreasing on $(0, 0.61]$; increasing on $[0.61, \infty)$

Not symmetric

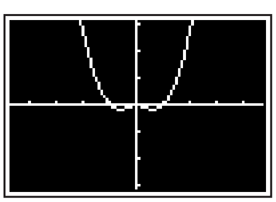
Bounded below

Local minimum at $(0.61, -0.18)$

No asymptotes

$\lim_{x\to\infty} f(x) = \infty$

41.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $[-0.18, \infty)$

Discontinuous at $x = 0$

Decreasing on $(-\infty, -0.61]$, $(0, 0.61]$;

Increasing on $[-0.61, 0)$, $[0.61, \infty)$

Symmetric across y-axis

Bounded below

Local minima at $(-0.61, -0.18)$
and $(0.61, -0.18)$

No asymptotes

$\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = \infty$

42.

$[0, 15]$ by $[-4, 1]$

Domain: $(0, \infty)$

Range: $\left(-\infty, \frac{1}{e}\right] \approx (-\infty, 0.37]$

Continuous

Increasing on $(0, e] \approx (0, 2.72]$,

Decreasing $[e, \infty) \approx [2.72, \infty)$

Not symmetric

Bounded above

Local maximum at $\left(e, \frac{1}{e}\right) \approx (2.72, 0.37)$

Asymptotes: $y = 0$ and $x = 0$

$\lim_{x\to\infty} f(x) = 0$

43. $x = \log 4 \approx 0.6021$

44. $x = \ln 0.25 = -1.3863$

45. $x = \frac{\ln 3}{\ln 1.05} \approx 22.5171$

46. $x = e^{5.4} = 221.4064$

47. $x = 10^{-7} = 0.0000001$

48. $x = 3 + \frac{\ln 5}{\ln 3} \approx 4.4650$

49. $\log_2 x = 2$, so $x = 2^2 = 4$

50. $\log_3 x = \frac{7}{2}$, so $x = 3^{7/2} = 27\sqrt{3} \approx 46.7654$

51. Multiply both sides by $2 \cdot 3^x$, leaving $(3^x)^2 - 1 = 10 \cdot 3^x$, or $(3^x)^2 - 10 \cdot 3^x - 1 = 0$. This is quadratic in 3^x, leading to $3^x = \frac{10 \pm \sqrt{100 + 4}}{2} = 5 \pm \sqrt{26}$. Only $5 + \sqrt{26}$ is positive, so the only answer is $x = \log_3(5 + \sqrt{26}) \approx 2.1049$.

52. Multiply both sides by $4 + e^{2x}$, leaving $50 = 44 + 11e^{2x}$, so $11e^{2x} = 6$. Then $x = \frac{1}{2} \ln \frac{6}{11} \approx -0.3031$.

53. $\log[(x + 2)(x - 1)] = 4$, so $(x + 2)(x - 1) = 10^4$.

The solutions to this quadratic equation are

$x = \frac{1}{2}(-1 \pm \sqrt{40{,}009})$, but of these two numbers, only the positive one, $x = \frac{1}{2}(\sqrt{40{,}009} - 1) \approx 99.5112$, works in the original equation.

54. $\ln \frac{3x+4}{2x+1} = 5$, so $3x + 4 = e^5(2x + 1)$.

Then $x = \frac{4 - e^5}{2e^5 - 3} \approx -0.4915$.

55. $\log_2 x = \frac{\ln x}{\ln 2}$

56. $\log_{1/6}(6x^2) = \log_{1/6} 6 + \log_{1/6} x^2 = \log_{1/6} 6 + 2\log_{1/6}|x|$

$= -1 + \frac{2\ln|x|}{\ln 1/6} = -1 + \frac{2\ln|x|}{\ln 6^{-1}} = -1 - \frac{2\ln|x|}{\ln 6}$

57. $\log_5 x = \frac{\log x}{\log 5}$

58. $\log_{1/2}(4x^3) = \log_{1/2} 4 + \log_{1/2} x^3 = -2 + 3\log_{1/2} x$

$= -2 - 3\log_2 x$

$= -2 - \frac{3\log x}{\log 2}$

59. Increasing, intercept at $(1, 0)$. The answer is (c).

60. Decreasing, intercept at $(1, 0)$. The answer is (d).

61. Intercept at $(-1, 0)$. The answer is (b).

62. Intercept at $(0, 1)$. The answer is (a).

63. $A = 450(1 + 0.046)^3 \approx \$515.00$

64. $A = 4800\left(1 + \frac{0.062}{4}\right)^{(4)(17)} \approx \$13{,}660.81$

65. $A = Pe^{rt}$

66. $i = \frac{r}{k}$, $n = kt$, so $FV = R \cdot \frac{\left(1 + \frac{r}{k}\right)^{kt} - 1}{\left(\frac{r}{k}\right)}$

67. $PV = \frac{550\left(1 - \left(1 + \frac{0.055}{12}\right)^{(-12)(5)}\right)}{\left(\frac{0.055}{12}\right)} \approx \$28{,}794.06$

68. $PV = \frac{953\left(1 - \left(1 + \frac{0.0725}{26}\right)^{(-26)(15)}\right)}{\left(\frac{0.0725}{26}\right)} \approx \$226{,}396.22$

69. $20e^{-3k} = 50$, so $k = -\frac{1}{3}\ln\frac{5}{2} \approx -0.3054$.

70. $20e^{-k} = 30$, so $k = -\ln\frac{3}{2} \approx -0.4055$.

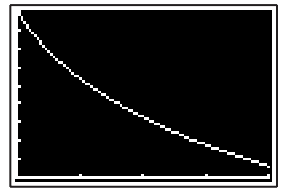
71. $P(t) = 2.0956 \cdot 1.01218^t$, where x is the number of years since 1900. In 2005, $P(105) = 2.0956 \cdot 1.01218^{105} \approx 7.5$ million.

72. $P(t) = \frac{14.3614}{(1 + 2.0083e^{-0.0249t})}$, where x is the number of years since 1900. In 2010, $P(110) \approx 12.7$ million.

73. **(a)** $f(0) = 90$ units.

(b) $f(2) \approx 32.8722$ units.

(c)



$[0, 4]$ by $[0, 90]$

74. **(a)** $P(t) = 123{,}000(1 - 0.024)^t = 123{,}000(0.976)^t$.

(b) $P(t) = 90{,}000$ when $t = \frac{\ln(90/123)}{\ln 0.976} \approx 12.86$ years.

75. **(a)** $P(t) = 89{,}000(1 - 0.018)^t = 89{,}000(0.982)^t$.

(b) $P(t) = 50{,}000$ when $t = \frac{\ln(50/89)}{\ln 0.982} \approx 31.74$ years.

76. **(a)** $P(0) \approx 5.3959$ — 5 or 6 students.

(b) $P(3) \approx 80.6824$ — 80 or 81 students.

(c) $P(t) = 100$ when $1 + e^{4-t} = 3$, or $t = 4 - \ln 2$ ≈ 3.3069 — sometime on the fourth day.

(d) As $t \to \infty$, $P(t) \to 300$.

77. **(a)** $P(t) = 20 \cdot 2^t$, where t is time in months. (Other possible answers: $20 \cdot 2^{12t}$ if t is in years, or $20 \cdot 2^{t/30}$ if t is in days).

(b) $P(12) = 81{,}920$ rabbits after 1 year. $P(60) \approx 2.3058 \times 10^{19}$ rabbits after 5 years.

(c) Solve $20 \cdot 2^t = 10{,}000$ to find $t = \log_2 500$ ≈ 8.9658 months — 8 months and about 29 days.

78. **(a)** $P(t) = 4 \cdot 2^t = 2^{t+2}$, where t is time in days.

(b) $P(4) = 64$ guppies after 4 days. $P(7) = 512$ guppies after 1 week.

(c) Solve $4 \cdot 2^t = 2000$ to find $t = \log_2 500 = 8.9658$ days — 8 days and about 23 hours.

79. **(a)** $S(t) = S_0 \cdot \left(\frac{1}{2}\right)^{t/1.5}$, where t is time in seconds.

(b) $S(1.5) = S_0/2$. $S(3) = S_0/4$.

(c) If $1\ g = S(60) = S_0 \cdot \left(\frac{1}{2}\right)^{60/1.5} = S_0 \cdot \left(\frac{1}{2}\right)^{40}$, then

$S_0 = 2^{40} \approx 1.0995 \times 10^{12}\ g = 1.0995 \times 10^9$ kg $= 1{,}099{,}500$ metric tons.

80. **(a)** $S(t) = S_0 \cdot \left(\frac{1}{2}\right)^{t/2.5}$, where t is time in seconds.

(b) $S(2.5) = S_0/2$. $S(7.5) = S_0/8$.

(c) If $1\ g = S(60) = S_0 \cdot \left(\frac{1}{2}\right)^{60/2.5} = S_0 \cdot \left(\frac{1}{2}\right)^{24}$, then

$S_0 = 2^{24} = 16{,}777{,}216$ g $= 16{,}777.216$ kg.

81. Let a_1 = the amplitude of the ground motion of the Feb 4 quake, and let a_2 = the amplitude of the ground motion of the May 30 quake. Then:

$$6.1 = \log\frac{a_1}{T} + B \quad\text{and}\quad 6.9 = \log\frac{a_2}{T} + B$$
$$\left(\log\frac{a_2}{T} + B\right) - \left(\log\frac{a_1}{T} + B\right) = 6.9 - 6.1$$
$$\log\frac{a_2}{T} - \log\frac{a_1}{T} = 0.8$$
$$\log\frac{a_2}{a_1} = 0.8$$
$$\frac{a_2}{a_1} = 10^{0.8}$$
$$a_2 \approx 6.31\, a_1$$

The ground amplitude of the deadlier quake was approximately 6.31 times stronger.

82. (a) Seawater:

$$-\log[H^+] = 7.6$$
$$\log[H^+] = -7.6$$
$$[H^+] = 10^{-7.6} \approx 2.51 \times 10^{-8}$$

Milk of Magnesia:

$$-\log[H^+] = 10.5$$
$$\log[H^+] = -10.5$$
$$[H^+] = 10^{-10.5} \approx 3.16 \times 10^{-11}$$

(b) $\dfrac{[H^+] \text{ of Seawater}}{[H^+] \text{ of Milk of Magnesia}} = \dfrac{10^{-7.6}}{10^{-10.5}} \approx 794.33$

(c) They differ by an order of magnitude of 2.9.

83. Solve $1500\left(1 + \dfrac{0.08}{4}\right)^{4t} = 3750$: $(1.02)^{4t} = 2.5$, so $t = \dfrac{1}{4}\dfrac{\ln 2.5}{\ln 1.02} \approx 11.5678$ years — round to 11 years 9 months (the next full compounding period).

84. Solve $12{,}500e^{0.09t} = 37{,}500$: $e^{0.09t} = 3$, so $t = \dfrac{1}{0.09}\ln 3 = 12.2068$ years.

85. $t = 133.83 \ln\dfrac{700}{250} \approx 137.7940$ — about 11 years 6 months.

86. $t = 133.83 \ln\dfrac{500}{50} \approx 308.1550$ — about 25 years 9 months.

87. $r = \left(1 + \dfrac{0.0825}{12}\right)^{12} - 1 \approx 8.57\%$

88. $r = e^{0.072} - 1 \approx 7.47\%$

89. $I = 12 \cdot 10^{(-0.0125)(25)} = 5.84$ lumens

90. $\log_b x = \dfrac{\ln x}{\ln b}$. This is a vertical stretch if $e^{-1} < b < e$ (so that $|\ln b| < 1$), and a shrink if $0 < b < e^{-1}$ or $b > e$. (There is also a reflection if $0 < b < 1$.)

91. $\log_b x = \dfrac{\log x}{\log b}$. This is a vertical stretch if $\dfrac{1}{10} < b < 10$ (so that $|\log b| < 1$), and a shrink if $0 < b < \dfrac{1}{10}$ or $b > 10$. (There is also a reflection if $0 < b < 1$.)

92. $g(x) = \ln[a \cdot b^x] = \ln a + \ln b^x = \ln a + x \ln b$. This has a slope $\ln b$ and y-intercept $\ln a$.

93. (a) $P(0) = 16$ students.

(b) $P(t) = 800$ when $1 + 99e^{-0.4t} = 2$, or $e^{0.4t} = 99$, so $t = \dfrac{1}{0.4}\ln 99 \approx 11.4878$ — about $11\frac{1}{2}$ days.

(c) $P(t) = 400$ when $1 + 99e^{-0.4t} = 4$, or $e^{0.4t} = 33$, so $t = \dfrac{1}{0.4}\ln 33 \approx 8.7413$ — about 8 or 9 days.

94. (a) $P(0) = 12$ deer.

(b) $P(t) = 1000$ when $1 + 99e^{-0.4t} = 1.2$, so $t = -\dfrac{1}{0.4}\ln\dfrac{0.2}{99} \approx 15.5114$ — about $15\frac{1}{2}$ years.

(c) As $t \to \infty$, $P(t) \to 1200$ (and the population never rises above that level).

95. The model is $T = 20 + 76e^{-kt}$, and $T(8) = 65 = 20 + 76e^{-8k}$. Then $e^{-8k} = \dfrac{45}{76}$, so $k = -\dfrac{1}{8}\ln\dfrac{45}{76} \approx 0.0655$. Finally, $T = 25$ when $25 = 20 + 76e^{-kt}$, so $t = -\dfrac{1}{k}\ln\dfrac{5}{76} \approx 41.54$ minutes.

96. The model is $T = 75 + 145e^{-kt}$, and $T(35) = 150 = 75 + 145e^{-35k}$. Then $e^{-35k} = \dfrac{75}{145}$, so $k = -\dfrac{1}{35}\ln\dfrac{15}{29} \approx 0.0188$. Finally, $T = 95$ when $95 = 75 + 145e^{-kt}$, so $t = -\dfrac{1}{k}\ln\dfrac{20}{145} \approx 105.17$ minutes.

97. (a) Matching up with the formula $S = R\dfrac{(1+i)^n - 1}{i}$, where $i = r/k$, with r being the rate and k being the number of payments per year, we find $r = 9\%$.

(b) $k = 4$ payments per year.

(c) Each payment is $R = \$100$.

98. (a) Matching up with the formula $A = R\dfrac{1 - (1+i)^{-n}}{i}$, where $i = r/k$, with r being the rate and k being the number of payments per year, we find $r = 11\%$.

(b) $k = 4$ payments per year.

(c) Each payment is $R = \$200$.

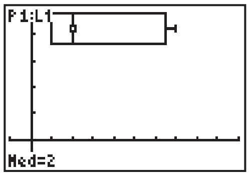
99. (a) Grace's balance will always remain \$1000, since interest is not added to it. Every year she receives 5% of that \$1000 in interest; after t years, she has been paid $5t\%$ of the \$1000 investment, meaning that altogether she has $1000 + 1000 \cdot 0.05t = 1000(1 + 0.05t)$.

(b) The table is shown below; the second column gives values of $1000e^{0.05t}$. The effects of compounding continuously show up immediately.

Years	Not Compounded	Compounded
0	1000.00	1000.00
1	1050.00	1051.27
2	1100.00	1105.17
3	1150.00	1161.83
4	1200.00	1221.40
5	1250.00	1284.03
6	1300.00	1349.86
7	1350.00	1419.07
8	1400.00	1491.82
9	1450.00	1568.31
10	1500.00	1648.72

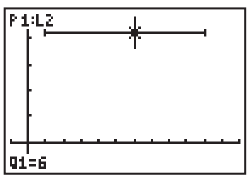
53. There are many possible answers; example data sets are given.

(a) $\{1, 1, 2, 6, 7\}$ — median $= 2$ and $\bar{x} = 3.4$.



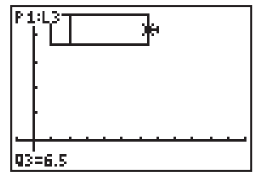
$[-1, 10]$ by $[-1, 5]$

(b) $\{1, 6, 6, 6, 6, 10\}$ — $2 \times \text{IQR} = 2(6 - 6) = 0$ and range $= 10 - 1 = 9$.



$[-1, 12]$ by $[-1, 5]$

(c) $\{1, 1, 2, 6, 7\}$ — range $= 7 - 1 = 6$ and $2 \times \text{IQR} = 2(6 - 1) = 10$.



$[-1, 12]$ by $[-1, 5]$

54. One possible answer: $\{1, 2, 3, 4, 5, 6, 6, 6, 30\}$.

55. For women living in South American nations, the mean life expectancy is

$$\bar{x} = \frac{(79.7)(39.1) + (67.9)(8.7) + \cdots + (79.2)(3.4) + (77.3)(25.0)}{39.1 + 8.7 + \cdots + 3.4 + 25.0} = \frac{27{,}825.56}{366.4} \approx 75.9 \text{ years.}$$

56. For men living in South American nations, the mean life expectancy is

$$\bar{x} = \frac{(72.0)(39.1) + (62.5)(8.7) + \cdots + (72.7)(3.4) + (71.0)(25.0)}{39.1 + 8.7 + \cdots + 3.4 + 25.0} = \frac{25{,}160.64}{366.4} \approx 68.7 \text{ years.}$$

57. Since $\sigma = 0.05$ mm, we have $2\sigma = 0.1$ mm, so 95% of the ball bearings will be acceptable. Therefore, 5% will be rejected.

58. Use $\mu = 12.08$ and $\sigma = 0.04$.

Then $\mu - 2\sigma = 12.00$ and $\mu + 2\sigma = 12.16$, so 95% of the cans contain 12 to 12.16 oz of cola, 2.5% contain less than 12 oz, and 2.5% contain more than 12 oz. Therefore, 2.5% of the cans contain less than the advertised amount.

Chapter 9 Review

1. $\binom{12}{5} = \frac{12!}{5!(12-5)!} = \frac{12!}{5!7!} = 792$

2. $\binom{789}{787} = \frac{789!}{787!(789-787)!} = \frac{789!}{787!2!} = 310{,}866$

3. ${}_{18}C_{12} = \frac{18!}{12!(18-12)!} = \frac{18!}{12!6!} = 18{,}564$

4. ${}_{35}C_{28} = \frac{35!}{28!(35-28)!} = \frac{35!}{28!7!} = 6{,}724{,}520$

5. ${}_{12}P_7 = \frac{12!}{(12-7)!} = \frac{12!}{5!} = 3{,}991{,}680$

6. ${}_{15}P_8 = \frac{15!}{(15-8)!} = \frac{15!}{7!} = 259{,}459{,}200$

7. $26 \cdot 36^4 = 43{,}670{,}016$ code words

8. $3 + (3 \cdot 4) = 15$ trips

9. ${}_{26}P_2 \cdot {}_{10}P_4 + {}_{10}P_3 \cdot {}_{26}P_3 = 14{,}508{,}000$ license plates

10. ${}_{45}C_3 = 14{,}190$ committees

11. Choose 10 more cards from the other 49: ${}_3C_3 \cdot {}_{49}C_{10} = {}_{49}C_{10} = 8{,}217{,}822{,}536$ hands

12. Choose a king, then 8 more cards from the other 44: ${}_4C_4 \cdot {}_4C_1 \cdot {}_{44}C_8 = {}_4C_1 \cdot {}_{44}C_8 = 708{,}930{,}508$ hands

13. ${}_5C_2 + {}_5C_3 + {}_5C_4 + {}_5C_5 = 2^5 - {}_5C_0 - {}_5C_1 = 26$ outcomes

14. ${}_{21}C_2 \cdot {}_{14}C_2 = 19{,}110$ committees

15. ${}_5P_1 + {}_5P_2 + {}_5P_3 + {}_5P_4 + {}_5P_5 = 325$

16. $2^4 = 16$ (This includes the possibility that he has *no* coins in his pocket.)

17. (a) There are 7 letters, all different. The number of distinguishable permutations is $7! = 5040$. (GERMANY can be rearranged to spell MEG RYAN.)

(b) There are 13 letters, where E, R, and S each appear twice. The number of distinguishable permutations is $\frac{13!}{2!2!2!} = 778{,}377{,}600$
(PRESBYTERIANS can be rearranged to spell BRITNEY SPEARS.)

18. (a) There are 7 letters, all different. The number of distinguishable permutations is $7! = 5040$.

(b) There are 11 letters, where A appears 3 times and L, S, and E each appear 2 times. The number of distinguishable permutations is $\frac{11!}{3!2!2!2!} = 831{,}600$

19.
$$\begin{aligned}(2x + y)^5 &= (2x)^5 + 5(2x)^4y + 10(2x)^3y^2 \\ &\quad + 10(2x)^2y^3 + 5(2x)y^4 + y^5 \\ &= 32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 \\ &\quad + 10xy^4 + y^5\end{aligned}$$

20. $(4a - 3b)^7 = (4a)^7 + 7(4a)^6(-3b) + 21(4a)^5(-3b)^2 + 35(4a)^4(-3b)^3 + 35(4a)^3(-3b)^4 + 21(4a)^2(-3b)5 + 7(4a)(-3b)^6 + (-3b)^7$
$= 16{,}384a^7 - 86{,}016a^6b + 193{,}536a^5b^2 - 241{,}920a^4b^3 + 181{,}440a^3b^4 - 81{,}648a^2b^5 + 20{,}412ab^6 - 2187b^7$

21. $(3x^2 + y^3)^5 = (3x^2)^5 + 5(3x^2)^4(y^3) + 10(3x^2)^3(y^3)^2 + 10(3x^2)^2(y^3)^3 + 5(3x^2)(y^3)^4 + (y^3)^5$
$= 243x^{10} + 405x^8y^3 + 270x^6y^6 + 90x^4y^9 + 15x^2y^{12} + y^{15}$

22. $\left(1 + \frac{1}{x}\right)^6 = 1 + 6(x^{-1}) + 15(x^{-1})^2 + 20(x^{-1})^3 + 15(x^{-1})^4 + 6(x^{-1})^5 + (x^{-1})^6$
$= 1 + 6x^{-1} + 15x^{-2} + 20x^{-3} + 15x^{-4} + 6x^{-5} + x^{-6}$

23. $(2a^3 - b^2)^9 = (2a^3)^9 + 9(2a^3)^8(-b^2) + 36(2a^3)^7(-b^2)^2 + 84(2a^3)^6(-b^2)^3 + 126(2a^3)^5(-b^2)^4 + 126(2a^3)^4(-b^2)^5 + 84(2a^3)^3(-b^2)^6 + 36(2a^3)^2(-b^2)^7 + 9(2a^3)(-b^2)^8 + (-b^2)^9$
$= 512a^{27} - 2304a^{24}b^2 + 4608a^{21}b^4 - 5376a^{18}b^6 + 4032a^{15}b^8 - 2016a^{12}b^{10} + 672a^9b^{12} - 144a^6b^{14} + 18a^3b^{16} - b^{18}$

24. $(x^{-2} + y^{-1})^4 = (x^{-2})^4 + 4(x^{-2})^3(y^{-1}) + 6(x^{-2})^2(y^{-1})^2 + 4(x^{-2})(y^{-1})^3 + (y^{-1})^4$
$= x^{-8} + 4x^{-6}y^{-1} + 6x^{-4}y^{-2} + 4x^{-2}y^{-3} + y^{-4}$

25. $\binom{11}{8}(1)^8(-2)^3 = -\frac{11!8}{8!3!} = -\frac{11 \cdot 10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = -1320$

26. $\binom{8}{2}(2)^2(1)^6 = \frac{8!4}{2!6!} = \frac{8 \cdot 7 \cdot 4}{2 \cdot 1} = 112$

27. {1, 2, 3, 4, 5, 6}

28. {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), ... , (6, 6)}

29. {13, 16, 31, 36, 61, 63}

30. {Defective, Nondefective}

31. {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

32. {HHT, HTH, THH, TTH, THT, HTT}

33. {HHH, TTT}

34. $P(\text{at least one head}) = 1 - P(\text{no heads}) = 1 - \left(\frac{1}{2}\right)^3 = \frac{7}{8}$

35. $P(\text{HHTHTT}) = \left(\frac{1}{2}\right)^6 = \frac{1}{2^6} = \frac{1}{64}$

36. $P(2\text{ H and }3\text{ T}) = {}_5C_2 \cdot \left(\frac{1}{2}\right)^5 = \frac{10}{2^5} = \frac{5}{16}$

37. $P(1\text{ H and }3\text{ T}) = {}_4C_1 \cdot \left(\frac{1}{2}\right)^4 = \frac{4}{2^4} = \frac{1}{4}$

38. $P(10\text{ nondefectives in a row}) = (0.997)^{10} \approx 0.97$

39. $P(3\text{ successes and 1 failure}) = {}_4C_1 \cdot \left(\frac{1}{2}\right)^4 = \frac{4}{2^4} = \frac{1}{4} = 0.25$

40. We can redefine the words "success" and "failure" to mean the opposite of what they meant before. In this sense, the experiment is symmetrical, because $P(\text{S}) = P(\text{F})$.

41. $P(\text{SF}) = (0.4)(0.6) = 0.24$

42. $P(\text{SFS}) = (0.4)(0.6)(0.4) = 0.096$

43. $P(\text{at least 1 success}) = 1 - P(\text{no successes}) = 1 - (0.6)^2 = 0.64$

44. Successes are less likely than failures, so the two are not interchangeable.

45. **(a)** $P(\text{brand }A) = 0.5$
(b) $P(\text{cashews from brand }A) = (0.5)(0.3) = 0.15$
(c) $P(\text{cashew}) = (0.5)(0.3) + (0.5)(0.4) = 0.35$
(d) $P(\text{brand }A/\text{cashew}) = \frac{0.15}{0.35} \approx 0.43$

46. **(a)** $P(\text{track wet and Mudder Earth wins}) = (0.80)(0.70) = 0.56$
(b) $P(\text{track dry and Mudder Earth wins}) = (0.20)(0.40) = 0.08$
(c) $0.56 + 0.08 = 0.64$
(d) $P(\text{track wet/Mudder Earth wins}) = \frac{0.56}{0.64} = 0.875$

For #47–48, substitute $n = 1, n = 2, \ldots, n = 6$, and $n = 40$.

47. 0, 1, 2, 3, 4, 5; 39

48. $-1, \frac{4}{3}, -2, \frac{16}{5}, -\frac{16}{3}, \frac{64}{7}; \approx 2.68 \times 10^{10}$

For #49–54, use previously computed values of the sequence to find the next term in the sequence.

49. −1, 2, 5, 8, 11, 14; 32

50. 5, 10, 20, 40, 80, 160; 10,240

51. −5, −3.5, −2, −0.5, 1, 2.5; 11.5

52. $3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}; 3^{-10} = \frac{1}{59{,}049}$

53. −3, 1, −2, −1, −3, −4; −76

54. −3, 2, −1, 1, 0, 1; 13

For #55–62, check for common difference or ratios between successive terms.

55. Arithmetic with $d = -2.5$;
$a_n = 12 + (-2.5)(n - 1) = 14.5 - 2.5n$

56. Arithmetic with $d = 4$;
$a_n = -5 + 4(n - 1) = 4n - 9$

57. Geometric with $r = 1.2$;
$a_n = 10 \cdot (1.2)^{n-1}$

58. Geometric with $r = -2$;
$a_n = \frac{1}{8} \cdot (-2)^{n-1} = -\frac{1}{16}(-2)^n$

59. Arithmetic with $d = 4.5$;
$a_n = -11 + 4.5(n - 1) = 4.5n - 15.5$

60. Geometric with $r = \frac{1}{4}$;
$b_n = 7 \cdot \left(\frac{1}{4}\right)^{n-1} = 28 \cdot \left(\frac{1}{4}\right)^n$

61. $a_n = a_1 r^{n-1}$, so $-192 = a_1 r^3$ and $196{,}608 = a_1 r^8$. Then $r^5 = -1024$, so $r = -4$, and $a_1 = \frac{-192}{(-4)^3} = 3$; $a_n = 3(-4)^{n-1}$

62. $a_n = a_1 + (n-1)d$, so $14 = a_1 + 2d$, and $-3.5 = a_1 + 7d$. Then $5d = -17.5$, so $d = -3.5$, and $a_1 = 14 - 2(-3.5) = 21$; $a_n = 21 - 3.5(n-1) = 24.5 - 3.5n$.

For #63–66, use one of the formulas $S_n = n\left(\frac{a_1 + a_n}{2}\right)$ or $S_n = \frac{n}{2}[2a_1 + (n-1)d]$. In most cases, the first of these is easier (since the last term a_n is given); note that $n = \frac{a_n - a_1}{d} + 1$.

63. $8 \cdot \left(\frac{-11 + 10}{2}\right) = 4 \cdot (-1) = -4$

64. $7 \cdot \left(\frac{13 - 11}{2}\right) = 7$

65. $27 \cdot \left(\frac{2.5 - 75.5}{2}\right) = \frac{1}{2} \cdot 27 \cdot (-73) = -985.5$

66. $31 \cdot \left(\frac{-5 + 55}{2}\right) = 31 \cdot 25 = 775$

For #67–70, use the formula $S_n = \frac{a_1(1 - r^n)}{1 - r}$. Note that $n = 1 + \log_{|r|}\left|\frac{a_n}{a_1}\right| = 1 + \frac{\ln|a_n/a_1|}{\ln|r|}$.

67. $\frac{4(1 - (-1/2)^6)}{1 - (-1/2)} = \frac{21}{8}$

68. $\frac{-3(1 - (1/3)^5)}{1 - (1/3)} = -\frac{121}{27}$

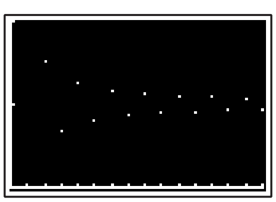
69. $\frac{2(1 - 3^{10})}{1 - 3} = 59{,}048$

70. $\frac{1(1 - (-2)^{14})}{1 - (-2)} = -5461$

71. Geometric with $r = \frac{1}{3}$:
$S_{10} = \frac{2187(1 - (1/3)^{10})}{1 - (1/3)} = \frac{29{,}524}{9} = 3280.\overline{4}$

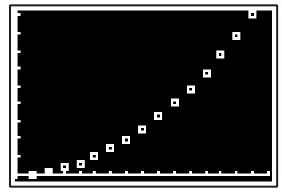
72. Arithmetic with $d = -3$:
$S_{10} = \frac{10}{2}[2(94) + 9(-3)] = 5 \cdot 161 = 805$

73.



[0, 15] by [0, 2]

74.



[0, 16] by [−10, 460]

75. With $a_1 = \$150$, $r = 1 + 0.08/12$, and $n = 120$, the sum becomes
$\frac{\$150\,[1 - (1 + 0.08/12)^{120}]}{1 - (1 + 0.08/12)} = \$27{,}441.91$

76. The payment amount P must be such that
$$P\left(1 + \frac{0.08}{12}\right)^0 + P\left(1 + \frac{0.08}{12}\right)^1 + \ldots + P\left(1 + \frac{0.08}{12}\right)^{119} \geq \$30{,}000$$
Using the formula for the sum of a finite geometric series,
$$\frac{P\,[1 - (1 + 0.08/12)^{120}]}{1 - (1 + 0.08/12)} \geq \$30{,}000$$
or $P \geq \$30{,}000 \frac{-0.08/12}{1 - (1 + 0.08/12)^{120}}$
$\approx \$163.983$
$\approx \$163.99$ rounded up

77. Converges: geometric with $a_1 = \frac{3}{2}$ and $r = \frac{3}{4}$, so $S = \frac{3/2}{1 - (3/4)} = \frac{3/2}{1/4} = 6$

78. Converges: geometric with $a_1 = -\frac{2}{3}$ and $r = -\frac{1}{3}$, so $S = \frac{-2/3}{1 - (-1/3)} = \frac{-2/3}{4/3} = -\frac{1}{2}$

79. Diverges: geometric with $r = -\frac{4}{3}$

80. Diverges: geometric with $r = \frac{6}{5}$

81. Converges: geometric with $a_1 = 1.5$ and $r = 0.5$, so $S = \frac{1.5}{1 - 0.5} = \frac{1.5}{0.5} = 3$

82. Diverges; geometric with $r = 1.2$

83. $\sum_{k=1}^{21}[-8 + 5(k-1)] = \sum_{k=1}^{21}(5k - 13)$

84. $\sum_{k=1}^{10} 4(-2)^{k-1} = \sum_{k=1}^{10}(-2)^{k+1}$

85. $\sum_{k=0}^{\infty}(2k+1)^2$ or $\sum_{k=1}^{\infty}(2k-1)^2$

86. $\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k$ or $\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{k-1}$

87. $\sum_{k=1}^{n}(3k+1) = 3\sum_{k=1}^{n}k + \sum_{k=1}^{n}1$
$= 3 \cdot \frac{n(n+1)}{2} + n = \frac{3n^2 + 5n}{2} = \frac{n(3n+5)}{2}$

88. $\sum_{k=1}^{n}3k^2 = 3\sum_{k=1}^{n}k^2 = 3 \cdot \frac{n(n+1)(2n+1)}{6}$
$= \frac{n(n+1)(2n+1)}{2}$