68. $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) =$

$$\langle u_1, u_2, u_3 \rangle \cdot \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$
$$= (u_1u_2v_3 - u_1u_3v_2) + (u_2u_3v_1 - u_1u_2v_3) + (u_1u_3v_2 - u_2u_3v_1)$$
$$= 0$$

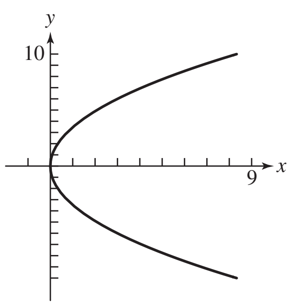
$\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) =$

$$\langle v_1, v_2, v_3 \rangle \cdot \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$
$$= (u_2v_1v_3 - u_3v_1v_2) + (u_3v_1v_2 - u_1v_2v_3) + (u_1v_2v_3 - u_2v_1v_3)$$
$$= 0$$

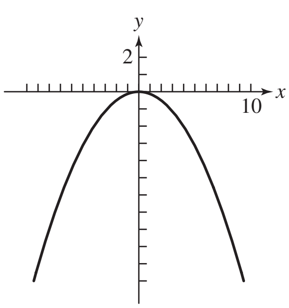
So the angles between **u** and **u** × **v**, and **v** and **u** × **v**, both have a cosine of zero by the theorem in Section 6.2. It follows that the angles both measure 90°.

■ Chapter 8 Review

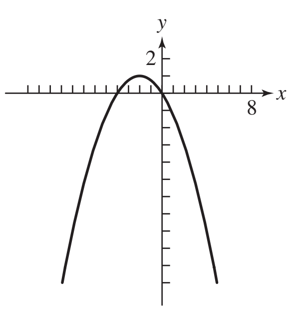
1. $h = 0, k = 0, 4p = 12$, so $p = 3$.
Vertex: (0, 0), focus: (3, 0), directrix: $x = -3$, focal width: 12



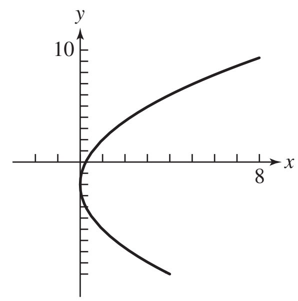
2. $h = 0, k = 0, 4p = -8$, so $p = -2$.
Vertex: (0, 0), focus: (0, −2), directrix: $y = 2$, focal width: 8



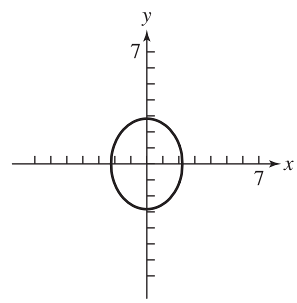
3. $h = -2, k = 1, 4p = -4$, so $p = -1$.
Vertex: (−2, 1), focus: (−2, 0), directrix: $y = 2$, focal width: 4



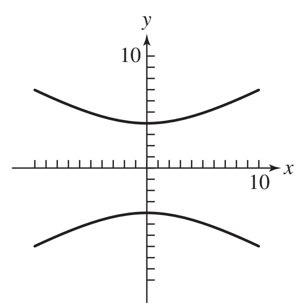
4. $h = 0, k = -2, 4p = 16$, so $p = 4$.
Vertex: (0, −2), focus: (4, −2), directrix: $x = -4$, focal width: 16



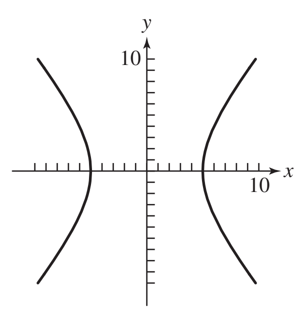
5. Ellipse. Center (0, 0). Vertices: $(0, \pm 2\sqrt{2})$. Foci: $(0, \pm\sqrt{3})$ since $c = \sqrt{8 - 5} = \sqrt{3}$.



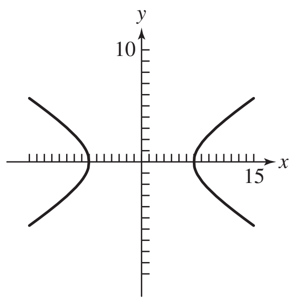
6. Hyperbola. Center: (0, 0). Vertices: (0, ±4).
Foci: $(0, \pm\sqrt{65})$ since $c = \sqrt{16 + 49} = \sqrt{65}$.



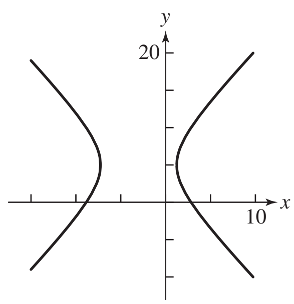
7. Hyperbola. Center: (0, 0). Vertices: (±5, 0),
$c = \sqrt{a^2 + b^2} = \sqrt{25 + 36} = \sqrt{61}$, so the foci are: $(\pm\sqrt{61}, 0)$



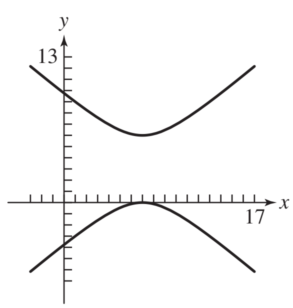
8. Hyperbola. Center: $(0, 0)$. Vertices: $(\pm 7, 0)$, $c = \sqrt{a^2 + 6^2} = \sqrt{49 + 9} = \sqrt{58}$, so the foci are: $(\pm\sqrt{58}, 0)$



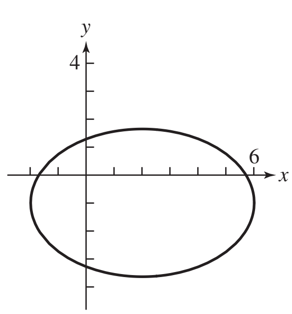
9. Hyperbola. Center: $(-3, 5)$. Vertices: $(-3 \pm 3\sqrt{2}, 5)$, $c = \sqrt{a^2 + b^2} = \sqrt{18 + 28} = \sqrt{46}$, so the foci are: $(-3 \pm \sqrt{46}, 5)$



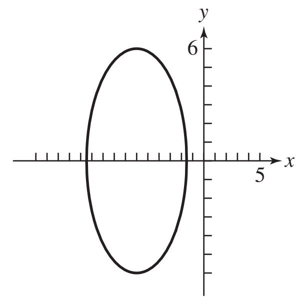
10. Hyperbola. Center: $(7, 3)$. Vertices: $(7, 3 \pm 3) = (7, 0)$ and $(7, 6)$, $c = \sqrt{a^2 + b^2} = \sqrt{9 + 12} = \sqrt{21}$, so the foci are: $(7, 3 \pm \sqrt{21})$



11. Ellipse. Center: $(2, -1)$. Vertices: $(2 \pm 4, -1) = (6, -1)$ and $(-2, -1)$, $c = \sqrt{a^2 - b^2} = \sqrt{16 - 7} = 3$, so the foci are: $(2 \pm 3, -1) = (5, -1)$ and $(-1, -1)$

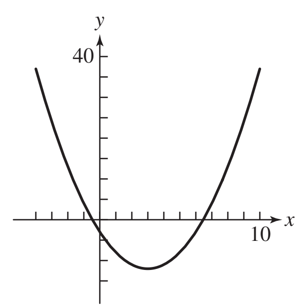


12. Ellipse. Center: $(-6, 0)$. Vertices: $(-6, \pm 6)$ $c = \sqrt{a^2 - b^2} = \sqrt{36 - 20} = 4$, so the foci are: $(-6, \pm 4)$

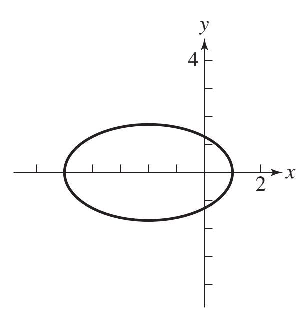


13. (b)
14. (g)
15. (h)
16. (e)
17. (f)
18. (d)
19. (c)
20. (a)

21. $B^2 - 4AC = 0 - 4(1)(0) = 0$, parabola $(x^2 - 6x + 9) = y + 3 + 9$, so $(x - 3)^2 = y + 12$

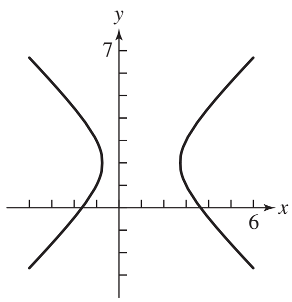


22. $B^2 - 4AC = 0 - 4(1)(3) = -12 < 0$, ellipse $(x^2 + 4x + 4) + 3y^2 = 5 + 4$, so $\dfrac{(x + 2)^2}{9} + \dfrac{y^2}{3} = 1$

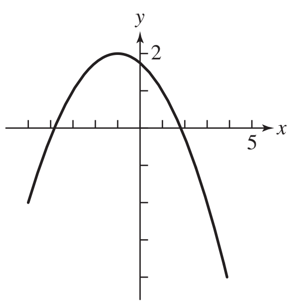


23. $B^2 - 4AC = 0 - 4(1)(-1) = 4 > 0$, hyperbola $(x^2 - 2x + 1) - (y^2 - 4y + 4) = 1 - 4 + 6$

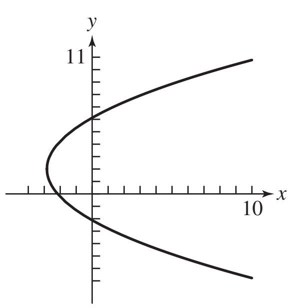
$$\frac{(x-1)^2}{3} - \frac{(y-2)^2}{3} = 1$$



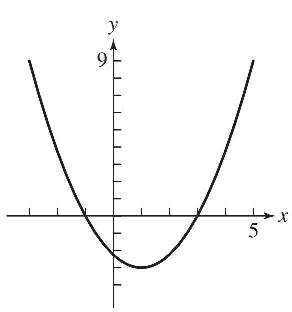
24. $B^2 - 4AC = 0 - 4(1)(0) = 0$, parabola $(x^2 + 2x + 1) = -4y + 7 + 1$, so $(x + 1)^2 = -4(y - 2)$



25. $B^2 - 4AC = 0 - 4(1)(0) = 0$, parabola $(y^2 - 4y + 4) = 6x + 13 + 4$, so $(y - 2)^2 = 6\left(x + \frac{17}{6}\right)$

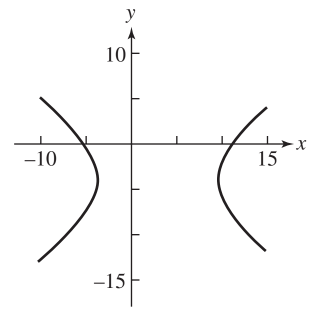


26. $B^2 - 4AC = 0 - 4(3)(0) = 0$, parabola $3(x^2 - 2x + 1) = 4y + 9 + 3$, so $(x - 1)^2 = \frac{4}{3}(y + 3)$



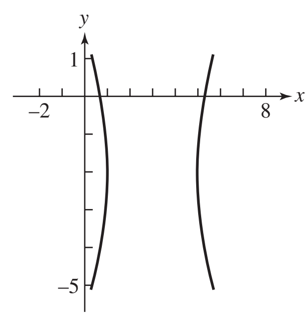
27. $B^2 - 4AC = 0 - 4(2)(-3) = 24 > 0$, hyperbola $2(x^2 - 6x + 9) - 3(y^2 + 8y + 16) = 18 - 48 - 60$, so

$$\frac{(y+4)^2}{30} - \frac{(x-3)^2}{45} = 1$$



28. $B^2 - 4AC = 0 - 4(12)(-4) = 192 > 0$, hyperbola $12(x^2 - 6x + 9) - 4(y^2 + 4y + 4) = 108 - 16 - 44$, so

$$\frac{(x-3)^2}{4} - \frac{(y+2)^2}{12} = 1$$



29. By definition, every point $P(x, y)$ that lies on the parabola is equidistant from the focus to the directrix. The distance between the focus and point P is: $\sqrt{(x-0)^2 + (y-p)^2} = \sqrt{x^2 + (y-p)^2}$, while the distance between the point P and the line $y = -p$ is: $\sqrt{(x-x)^2 + (y+p)^2} = \sqrt{(y+p)^2}$. Setting these equal:

$$\sqrt{x^2 + (y-p)^2} = y + p$$
$$x^2 + (y-p)^2 = (y+p)^2$$
$$x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$$
$$x^2 = 4py$$

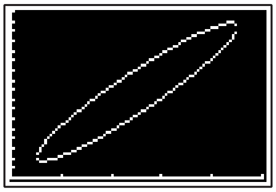
30. Let the point $P(x, y)$ satisfy $y^2 = 4px$. Then we have

$$y^2 = 4px$$
$$x^2 - 2px + p^2 + y^2 = x^2 + 2px + p^2$$
$$(x-p)^2 + y^2 = (x+p)^2 + 0$$
$$(x-p)^2 + (y-0)^2 = (x-(-p))^2 + (y-y)^2$$
$$\sqrt{(x-p)^2 + (y-0)^2} = \sqrt{(x-(-p))^2 + (y-y)^2}$$

distance from $P(x, y)$ to $(p, 0)$ = distance from $P(x, y)$ to $x = -p$

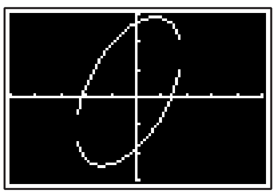
Because $P(x, y)$ is equidistant from the point $(p, 0)$ and the line $x = -p$, by the definition of a parabola, $y^2 = 4px$ is the equation of a parabola with focus $(p, 0)$ and directrix $x = -p$.

31. Use the quadratic formula with $a = 6, b = -8x - 5$, and $c = 3x^2 - 5x + 20$. Then $b^2 - 4ac = (-8x - 5)^2 - 24(3x^2 - 5x + 20) = -8x^2 + 200x - 455$, and $y = \frac{1}{12}\left[8x + 5 \pm \sqrt{-8x^2 + 200x - 455}\right]$ – an ellipse



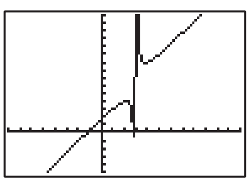
$[0, 25]$ by $[0, 17]$

32. Use the quadratic formula with $a = 6, b = -8x - 5$, and $c = 10x^2 + 8x - 30$. Then $b^2 - 4ac = (-8x - 5)^2 - 24(10x^2 + 8x - 30) = -176x^2 - 112x + 745$, and $y = \frac{1}{12}\left[8x + 5 \pm \sqrt{-176x^2 - 112x + 745}\right]$ an ellipse



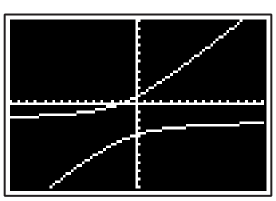
$[-5, 5]$ by $[-3, 3]$

33. This is a *linear* equation in y: $(6 - 2x)y + (3x^2 - 5x - 10) = 0$. Subtract $3x^2 - 5x - 10$ and divide by $6 - 2x$, and we have $y = \frac{3x^2 - 5x - 10}{2x - 6}$ — a hyperbola.



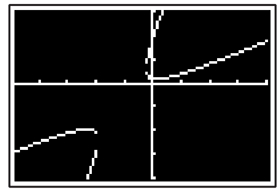
$[-8, 12]$ by $[-5, 15]$

34. Use the quadratic formula with $a = -6, b = 5x - 17$, and $c = 10x + 20$. Then $b^2 - 4ac = (5x - 17)^2 + 24(10x + 20) = 25x^2 + 70x + 769$, and $y = \frac{1}{12}\left[5x - 17 \pm \sqrt{25x^2 + 70x + 769}\right]$ a hyperbola.



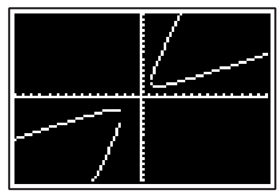
$[-15, 15]$ by $[-10, 10]$

35. Use the quadratic formula with $a = -2, b = 7x + 20$, and $c = -3x^2 - x - 15$. Then $b^2 - 4ac = (7x + 20)^2 + 8(-3x^2 - x - 15) = 25x^2 + 272x + 280$, and $y = \frac{1}{4}\left[7x + 20 \pm \sqrt{25x^2 + 272x + 280}\right]$ a hyperbola.



$[-24, 20]$ by $[-20, 15]$

36. Use the quadratic formula with $a = -2, b = 7x + 3$, and $c = -3x^2 - 2x - 10$. Then $b^2 - 4ac = (7x + 3)^2 + 8(-3x^2 - 2x - 10) = 25x^2 + 26x - 71$, and $y = \frac{1}{4}\left[7x + 3 \pm \sqrt{25x^2 + 26x - 71}\right]$ a hyperbola.



$[-15, 15]$ by $[-15, 15]$

37. $h = 0, k = 0, p = 2$, and the parabola opens to the right as $y^2 = 8x$.

38. $h = 0, k = 0, |4p| = 12$, and the parabola opens downward, so $x^2 = -12y$ $(p = -3)$.

39. $h = -3, k = 3, p = k - y = 3 - 0 = 3$ (since $y = 0$ is the directrix) the parabola opens upward, so $(x + 3)^2 = 12(y - 3)$.

40. $h = 1, k = -2, p = 2$ (since the focal length is 2), and the parabola opens to the left, so $(y + 2)^2 = -8(x - 1)$.

41. $h = 0, k = 0, c = 12$ and $a = 13$, so $b = \sqrt{a^2 - c^2} = \sqrt{169 - 144} = 5$. $\frac{x^2}{169} + \frac{y^2}{25} = 1$

42. $h = 0, k = 0, c = 2$ and $a = 6$, so $b = \sqrt{a^2 - c^2} = \sqrt{36 - 4} = 4\sqrt{2}$. $\frac{y^2}{36} + \frac{x^2}{32} = 1$

43. $h = 0, k = 2, a = 3, c = 2 - h$ (so $c = 2$) and $b = \sqrt{a^2 - c^2} = \sqrt{9 - 4} = \sqrt{5}$

$$\frac{x^2}{9} + \frac{(y - 2)^2}{5} = 1$$

44. $h = -3, k = -4, a = 4, 0 = -3 \pm c, c = 3$, $b = \sqrt{a^2 - c^2} = \sqrt{16 - 9} = \sqrt{7}$, so

$$\frac{(x + 3)^2}{16} + \frac{(y + 4)^2}{7} = 1$$

45. $h = 0, k = 0, c = 6, a = 5$, $b = \sqrt{c^2 - a^2} = \sqrt{36 - 25} = \sqrt{11}$, so

$$\frac{y^2}{25} - \frac{x^2}{11} = 1$$

46. $h = 0, k = 0, a = 2, \frac{b}{a} = 2$ $(b = 4)$, so $\frac{x^2}{4} - \frac{y^2}{16} = 1$