Precalculus Honors
Review 3.5, 3.6 – thought provokers

1. You have just won a lottery, and are offered one of two prizes: you can either receive $10,000 per month for the next 30 years, or else get a one-time payment of $2 million immediately. Assume that interest rates are currently at 2.9%.

SOLVER
N = 360
I% = 2.9
PV = 0
PMT = -10000
FV = ?
P/Y = 12
C/Y = 12

a) If you were to keep the money in the bank until the end of the 30 years, which option would be worth more at the end of that period? How much more?

ANNUITY (FV)
5,728,580.54

LUMP
$4,768,813.93 = 2,000,000\left(1+\frac{0.029}{12}\right)^{12(30)}$

DIFF = $959,766.61
MORE VALUE IN ANNUITY

b) Which option would you choose? Why? What factors influenced your decision?

ANSWERS VARY

- PLANS
- AGE
- TAX BRACKET
- INFLATION
- OTHER INVESTMENT OPTIONS

c) How much more attractive would the other option have to be to change your mind? (For example, if you chose the lump sum, how large would the monthly payments have to be to convince you to opt for those instead?)

ANSWERS VARY

2) You want to take out a loan for $125,000 and interest rates are at 5%, for how long would you have to make monthly payments if you can only afford to pay $500 per month? (What calculations did you make to arrive at your answer? Can you explain why you got this result?)

SOLVER
N = ?
I% = 5
PV = 125,000
PMT = -500
FV = 0
P/Y = 12
C/Y = 12

→ DOMAIN ERROR

NOT POSSIBLE. THE PV WILL NEVER REACH $125,000 FOR THIS SCENARIO. ($125,000 WOULD GENERATE MORE THAN $500 INTEREST EACH MONTH, SO YOUR PAYMENTS WOULD NEVER PAY IT OFF.)

2. A hot potato is placed on a table in a 68° F room. If the potato cooled from its initial toasty temperature of 250° F to a mere 130° F in only 12 minutes, how long would it take for the potato to cool to 78° F?

Possibly helpful formula: $T(t) = T_m + (T_0 - T_m)e^{-kt}$

t	T	T − 68°
0	250	182
12	130	62
?	78	10

$y = a \cdot b^t$

$y = 182\, b^t$

$62 = 182\, b^{12}$

$b^{12} = 0.341$

$b \approx 0.914$

$y = 182(0.914)^t$

$10 = 182(0.914)^t$

$t \approx$ **32.33 min**

3. An earthquake rips through Wilsonopolis with a vengeance. This quake was 22 times as intense as the last earthquake that had hit the city a decade earlier. If the previous quake had a Richter scale measurement of 4.1 on the Richter scale, what score would this more recent earthquake have earned?

$R_1 = 4.1$

$R_2 = ?$

$10^{(R_2 - 4.1)} = 22$

$R_2 - 4.1 = \log 22$

$R_2 = 4.1 + \log 22$

$R_2 =$ **5.44**

Key

Precalculus Honors 3.5 Solving Equations

Solve for x.

1) $300 = 100\left(1+\frac{0.05}{12}\right)^{12x}$

22.02

2) $x = 50\left(1+\frac{0.05}{4}\right)^{4(10)}$

82.18

3) $2{,}000 = x\left(1+\frac{0.05}{365}\right)^{365(50)}$

164.20

4) $5 = 1\left(1+\frac{x}{52}\right)^{52(10)}$

0.1612

5) $x = 50\left(\frac{\left(1+\frac{0.06}{4}\right)^{4(12)}-1}{\frac{0.06}{4}}\right)$

3,478.26

6) $100{,}000 = 500\left(\frac{1-\left(1+\frac{0.05}{12}\right)^{-12x}}{\frac{0.05}{12}}\right)$

35.91

$50{,}000 = x\left(\frac{1-\left(1+\frac{0.04}{12}\right)^{-12(10)}}{\frac{0.04}{12}}\right)$

506.23

note: 5, 9, 11 are more challenging!

PreCalc Honors – Review §§3.5 **Name** KEY
NO CALCULATORS **Period** _____

Write each expression in the form indicated.

1. Write $\log\sqrt[5]{\frac{x^2}{y^3}}$ in terms of $\log x$ and $\log y$

$= \log\left(\frac{x^2}{y^3}\right)^{\frac{1}{5}} = \frac{1}{5}\log\left(\frac{x^2}{y^3}\right) = \frac{1}{5}(\log x^2 - \log y^3) = \frac{1}{5}(2\log x - 3\log y)$

$= \frac{2}{5}\log x - \frac{3}{5}\log y$

2. Write $4\ln a + \frac{\ln b}{2} - 3\ln c$ as a single logarithm.

$\ln a^4 + \frac{1}{2}\ln b - \ln c^3$

$= \ln a^4 + \ln b^{\frac{1}{2}} - \ln c^3 = \ln\left(\frac{a^4 b^{\frac{1}{2}}}{c^3}\right)$

Simplify each expression. SHOW ALL WORK!

3. $\log_{\frac{1}{27}} 9 = \frac{\log_3 9}{\log_3\left(\frac{1}{27}\right)} = \frac{2}{-3} = -\frac{2}{3}$

4. $4\ln\sqrt{e} = 4\ln e^{1/2} = 4\left(\frac{1}{2}\right) = 2$

5. $16^{\log_8 3} = x$

$\left(8^{\frac{4}{3}}\right)^{\log_8 3} = \left(8^{\log_8 3}\right)^{\frac{4}{3}}$

$3^{\frac{4}{3}} = \sqrt[3]{81}$ or $3\sqrt[3]{3}$

6. $e^{4\ln 5}$

$= e^{\ln 5^4}$

$= 5^4$

$= 625$

7. $\log_3 8 \cdot \log_{64}\frac{1}{27}$

$= \frac{\log 8}{\log 3} \cdot \frac{\log\left(\frac{1}{27}\right)}{\log 64} = \frac{\log 8}{\log 3} \cdot \frac{-3\log 3}{2\log 8} = -\frac{3}{2}$

8. $\log 32 + 2\log 5 - 3\log 2$

$\log 2^5 + \log 5^2 - \log 2^3$

$\log\left(\frac{2^5 \cdot 5^2}{2^3}\right) = \log(2^2 5^2)$

$= \log 10^2$

$= 2$

9. $\frac{1}{\log_{45} 15} + \frac{1}{2\log_{25} 15}$

$= \frac{\log 45}{\log 15} + \frac{\log 25}{2\log 15} = \frac{\log 45 + \frac{1}{2}\log 25}{\log 15}$

$= \frac{\log 45 + \log 5}{\log 15} = \frac{\log 225}{\log 15} = \log_{15} 225 = 2$

10. $2^{3+\log_2 7}$

$= 2^3 2^{\log_2 7}$

$= 8 \cdot 7 = 56$

11. $3^{\log_{27} 8}$

$3^{\frac{\log 8}{\log 27}} = 3^{\frac{3\log 2}{3\log 3}} = 3^{\log_3 2} = 2$

12. $\frac{\log_7 64}{\log_{\frac{1}{7}} 16}$

$= \frac{\left(\frac{\log 64}{\log 7}\right)}{\left(\frac{\log 16}{\log\left(\frac{1}{7}\right)}\right)} = \frac{\log 64}{\log 7} \cdot \frac{\log\left(\frac{1}{7}\right)}{\log 16}$

$= \frac{6\log 2}{\log 7} \cdot \frac{-1\log 7}{4\log 2} = \frac{6(-1)}{4} = -\frac{3}{2}$

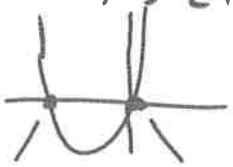
Honors Precalculus
Review §§3.5-3.6

Name _____
Period _____

Calculators Allowed

$$A = P\left(1+\frac{r}{n}\right)^{nt} \qquad FV = R\frac{(1+i)^n - 1}{i} \qquad PV = R\frac{1-(1+i)^{-n}}{i}$$

[-10,10] x [-10,10]



-4.24 0.236

1. Determine the domain of $f(x) = \log\left(x^2+4x-1\right)$. Show Work!

$x^2+4x-1 > 0$

← (graph)

$(-\infty, -4.24) \cup (0.24, \infty)$

2. A metal ingot is heated to a temperature of 500° F and then placed in a 70° F room to cool. After 4 minutes the bar has a temperature of 380° F. If the bar is considered safe to handle once the temperature cools down to 110° F, how long must the bar remain in the room (total) before it is safe to handle?

t	T	T-70
0	500	430
4	380	310
?	110	40
		(y)

$y = 430b^t$

$310 = 430b^4$

$b = \sqrt[4]{\frac{310}{430}} = 0.9215$

$40 = 430(0.9215)^t$

$0.0930 = (0.9215)^t$

$\log 0.0930 = t \log 0.9215$

$t = \frac{\log 0.0930}{\log 0.9215}$

$\approx$ 29.05 minutes

3. What is the annual percentage yield (APY) of an account paying an annual interest rate of 4.2%, compounded monthly?

$\left(1+\frac{0.042}{12}\right)^{12} = 1.042818$ (multiplier for 1 year)

APY = 4.2818%

4. Every week, Norbert puts $10 of his bottle-return money into a bank account earning 6.25% interest (compounded weekly). If he continues to do this for 20 years, how much will his account be worth? (assume 52 weeks per year)

$FV = 10\,\frac{\left(1+\frac{0.0625}{52}\right)^{20\cdot 52} - 1}{\frac{0.0625}{52}} = \$20,697.86$

5. Bert's brother wants to buy a new 10-speed bike which costs $480. He takes out a 30-year bike loan with an annual interest rate of 18% (compounded monthly). How much are his monthly payments?

$(PV)\ 480 = R\,\frac{1-\left(1+\frac{0.18}{12}\right)^{-30\cdot 12}}{\frac{0.18}{12}} = R(66.3532)$

$R = \frac{480}{66.35} = \$7.23$