PreCalculus Honors
9.4 Series **Calculators Allowed**

Name KEY
Period ______

1. Find the sum of the first thirty terms of -2, 3, 8, …

$n = 30$, $a_1 = -2$, $d = 3-(-2) = 5$ so…

$S_{30} = \frac{30}{2}\left[2(-2)+5(30-1)\right]$

$= 15\left[-4+145\right] = \boxed{2{,}115}$

2. Find the sum of the first twenty positive multiples of 3.

$= 3(1)+3(2)+3(3)+\dots+3(20)$

$= 3+6+9+\dots+60$

$n = 20$, $a_1 = 3$, $d = 3$ so…

$S_{20} = \frac{20}{2}\left[2(3)+3(20-1)\right]$

$= 10\left[6+57\right] = \boxed{630}$

3. Find the sum of the series. $\sum_{k=1}^{25} 7-2k$ $= 5+3+1+\dots+(-43)$

$a_1 = 5$, $d = 3-5 = -2$, $n = 25$

$S_{25} = \frac{25}{2}\left[2(5)+(-2)(25-1)\right] = \boxed{-475}$

4. How many terms of -10, -7, -4, … must be added to give a sum of 200?

$a_1 = -10$, $S_n = 200$, $d = -7-(-10) = 3$ so $S_n = \frac{n}{2}\left[2(-10)+3(n-1)\right] = 200$

$\frac{n}{2}\left[-20+3n-3\right] = 200$

$3n^2-23n-400 = 0$

$(3n+25)(n-16) = 0$

$n = \cancel{-\frac{25}{3}}$ or $\boxed{n = 16}$

(you can solve this by graphing if you prefer)

5. Find the sum of all positive integers less than 500 that are multiples of 11.

$500 \div 11 \approx 45.454\ldots \rightarrow 11(45) = 495$ is the largest

$11+22+33+44+\dots+495$

$n = 45$, $a_1 = 11$, $d = 11$ so…

$S_{45} = \frac{45}{2}\left[2(11)+11(45-1)\right] = \boxed{11{,}385}$

6. If $t_4 = ½$ and $t_9 = \frac{1}{64}$, find the sum of the first 12 terms of the geometric series.

$n = 12$

$\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underset{4}{\underline{½}} \rightarrow \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underset{9}{\underline{1/64}}$

$\frac{1}{2} \cdot r^5 = \frac{1}{64}$

$r^5 = \frac{1}{32}$

$r = ½$

working backwards,

$t_3 = 1$

$t_2 = 2$

$t_1 = 4$

$S_{12} = \frac{4\left(1-\left(\frac{1}{2}\right)^{12}\right)}{1-½}$

$\approx \boxed{7.998}$

7. Find the common ratio in a geometric sequence if $a_1 = -8$ and $S_3 = -8$.

$S_3 = \frac{a_1(1-r^3)}{1-r}$ so… $\frac{-8(1-r^3)}{1-r} = -8$

$-8(1-r^3) = -8(1-r)$

$1-r^3 = 1-r$

$r^3 - r = 0$

$r(r^2-1) = 0$

$r(r+1)(r-1) = 0$

$r = 0, -1, \cancel{1}$ ← extraneous

$\boxed{\{0, -1\}}$

Check: $r = 0 \Rightarrow -8+0+0 = -8$ ✓

$r = -1 \Rightarrow -8+8+-8 = -8$ ✓

$r = 1 \Rightarrow -8+-8+-8 \neq -8$ ✗

8. Find the seventh term in a geometric sequence for which $r = \frac{1}{2}$ and $S_7 = \dfrac{381}{4}$

$$S_7 = \frac{a_1\left(1-\left(\frac{1}{2}\right)^7\right)}{\left(1-\frac{1}{2}\right)} = \frac{381}{4}$$

$$a_1 = \frac{381}{8} \cdot \frac{128}{127} = 48$$

$$a_1\left(1-\frac{1}{128}\right) = \frac{381}{8}$$

$$a_7 = 48\left(\tfrac{1}{2}\right)^6 = \boxed{3/4} \text{ or } \boxed{0.75}$$

$$a_1\left(\frac{127}{128}\right) = \frac{381}{8}$$

9. Find Sn (the sum of the first n terms) for a geometric sequence in which $a_1 = 75$, $r = 1.4$, and $a_n = 288.12$

$$a_n = a_1 (r)^{n-1}$$

$$288.12 = 75(1.4)^{n-1}$$

$$S_5 = \frac{75\left(1-1.4^5\right)}{1-1.4} = \boxed{820.92}$$

$$3.8416 = 1.4^{n-1}$$

OR

SOLVE BY GRAPHING

$$\log 3.8416 = (n-1)\log 1.4$$

$$n-1 = \frac{\log 3.8416}{\log 1.4} = 4$$

So... $n=5$

10. Find the sum of the infinite geometric series: $\sum_{k=1}^{\infty} 8\left(-\frac{1}{2}\right)^{k-1}$ $= 8 + (-4) + 2 + (-1) + \frac{1}{2} + \ldots$

so $a_1 = 8$

$r = -\frac{1}{2}$

$$S_\infty = \frac{8}{1+\frac{1}{2}} = \frac{8}{\left(\frac{3}{2}\right)} = \boxed{\frac{16}{3}}$$

11. Find the sum of the infinite geometric series: $35 - \dfrac{35}{\sqrt{6}} + \dfrac{35}{6} - \ldots$

$a_1 = 35$

$$r = \frac{\left(\frac{-35}{\sqrt{6}}\right)}{35} = -\frac{1}{\sqrt{6}}$$

$$S_\infty = \frac{35}{1-\left(-\frac{1}{\sqrt{6}}\right)} = \boxed{\frac{35}{1+\frac{1}{\sqrt{6}}}} \text{ or } \boxed{\frac{35\sqrt{6}}{\sqrt{6}+1}} \text{ or } \boxed{42-7\sqrt{6}}$$

or $\approx \boxed{24.854}$

12. Write the first three terms of the infinite geometric sequence for which $r = -\dfrac{3}{4}$ and $S_\infty = 16$

$$S_\infty = \frac{a_1}{1-r}$$

$$a_2 = 28\left(-\frac{3}{4}\right) = -21$$

$$a_3 = -21\left(-\frac{3}{4}\right) = \frac{63}{4}$$

$$16 = \frac{a_1}{1-(-3/4)}$$

$$16 = \frac{a_1}{1+3/4}$$

$$\boxed{28, \ -21, \ \frac{63}{4}}$$

$$16 = \frac{a_1}{(7/4)} \rightarrow a_1 = 16(7/4) = 28$$